RELIABILITY ANALYSIS AND DESIGN OF REINFORCED BRICK BEAMS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

By

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to the

DEPARTMENT OF CIVIL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MAY, 1983

DEDICATED

TO

MY PARENTS

CERTIFICATE

This is to certify that the thesis entitled 'Reliability Analysis and Design of Reinforced Brick Beams' submitted by Sri Sitangshu Mukhopadhyay in partial fulfilment of the requirements for the degree of Doctor of Philosophy of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out under my supervision and has not been submitted elsewhere for a degree.

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Sitangshu Mukhopadhyay

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A	=	area of brick
A_{st}	=	area of steel
A_{stb}	=	area of steel for a balanced section
A_{std}	=	area of steel by limit state design
B	=	breadth of brick
C_k	=	coefficient of kurtosis
C_s	=	coefficient of skewness
D	=	dead load, diameter of reinforcing bar
E	=	Young's modulus of brickwork
E_s	=	Young's modulus of steel
F_D	=	partial safety factor applied to mean dead load moment
F_L	=	partial safety factor applied to mean live load moment
$F_X(x)$	=	probability distribution function of random variable X. Similar definitions for $F_{M_e}(x)$, $F_{M_r}(x)$ etc.
H	=	height of brick
L	=	length of brick, live load
M_D	=	dead load moment
M_{Dm}	=	mean dead load moment
M_L	=	live load moment
M_{Lm}	=	mean live load moment
M_d	=	design moment

M_e	=	external moment
M_{em}	=	mean of M_e
M_r	=	moment capacity
M_{rm}	=	mean of M_r
M_{ro}	=	over-reinforced moment capacity
M_{rom}	=	mean of M_{ro}
M_{ru}	=	under-reinforced moment capacity
M_{rum}	=	mean of M_{ru}
M_{rust}	=	mass of rust
M_{st}	=	mass of steel
$N(\mu, \sigma)$	=	normal density function with mean μ and standard deviation σ
P	=	compressive load
P(E)	=	probability of the occurrence of event E
R	=	resistance
S	=	load
V	=	volume of brick
V_r	=	volume expansion ratio
W_d	=	dry weight of brick
W_s	=	saturated weight of brick
X	=	a random variable
a	=	depth of stress block
b	=	breadth of RBB section
d	=	effective depth
d_c	=	cover thickness

f_b	=	strength of brick
f_{bm}	=	mean of f_b
f_m	=	strength of mortar
f_{mk}	=	characteristic strength of mortar
f_{mm}	=	mean of f_m
f_w	=	strength of masonry
f_{wk}	=	characteristic strength of masonry
f_{wm}	=	mean of f_w
f_{wt}	=	average tensile strength of brickwork
$F_X(x)$	=	probability density function of random variable X. Similar definitions for $f_{M_e}(x)$, $f_{M_r}(x)$ etc.
f_u	=	ultimate strength of steel
f_y	=	yield or proof strength of steel
f_{yk}	=	characteristic strength of steel
f_{ym}	=	mean of f_y
$g(t)$	=	deterioration function
j_r	=	average penetration rate per unit time
j_{cor}	=	rate at which the steel is converted to rust
k	=	number of classes in chi-square test
n	=	number of sample
p	=	percent of water absorption of brick
p_f	=	probability of failure
p_{fo}	=	probability of failure as an over-reinforced
p_{fu}	=	probability of failure as an under-reinforced
p-level	=	maximum significance level
p_k	=	characteristic p-level

p_m = mean of p-level

p_r = pressure developed due to corrosion

s = spacing of reinforcement

s_b = standard deviation of f_b

s_m = standard deviation of f_m

s_w = standard deviation of f_w

s_y = standard deviation of f_y

s_{M_D} = standard deviation of M_D

s_{M_L} = standard deviation of M_L

s_{M_e} = standard deviation of M_e

s_{M_r} = standard deviation of M_r

s_X = standard deviation of random variable X

t = time

t_p = depassivation time

t_{cr} = time of cracking or spalling of brickwork

t_{cor} = duration of steady state corrosion

α = significance level

α_{max} = maximum significance level

β = reliability index

γ = strength reduction factor

γ_d = dry density of brick

γ_w = wet density of brick, partial safety factor applied to strength of masonry

γ_y = partial safety factor applied to strength of steel

δ, δ_X	=	coefficient of variation of X
δ_{M_D}	=	coefficient of variation of M_D
δ_{M_L}	=	coefficient of variation of M_L
δ_{M_e}	=	coefficient of variation of M_e
δ_{M_r}	=	coefficient of variation of M_r
$\delta_{M_{ro}}$	=	coefficient of variation of M_{ro}
$\delta_{M_{ru}}$	=	coefficient of variation of M_{ru}
δ_{fy}	=	coefficient of variation of f_y
δ_{fw}	=	coefficient of variation of f_w
ϵ_{wmax}	=	maximum strain in brickwork
ϵ_s	=	strain in steel
ν	=	Poisson's ratio, degrees of freedom, central safety factor
ρ_{rust}	=	density of rust
ρ_{st}	=	density of steel
σ_{ln}	=	parameter of lognormal distribution
$\emptyset(x)$	=	cumulative distribution function of standard normal random variable
	=	$\int_{-\infty}^{x} e^{-z^2/2}\, dz$
$\emptyset^{-1}(.)$	=	inverse function of $\emptyset(.)$
χ^2	=	chi-square
$\{\ldots\}$	=	an event

C.O.V.	=	coefficient of variation
HYSD bar	=	high yield strength deformed bar
RBB	=	reinforced brick beam
RBM	=	reinforced brick masonry
S.D.	=	standard deviation.

SYNOPSIS

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Strength of reinforced brickwork is influenced by several variables such as size and properties of brick, properties of mortar and reinforcement, and proportions of the section. A large number of brick samples were studied for the statistical variations in their dimensional and strength properties. Dimensions of bricks manufactured from a single manufacturer have a very small variability with a tendency towards deterministic value. The coefficient of variation in length, breadth and height of bricks was limited to 3.4 percent indicating a high consistency in the dimensional tolerance of the hand made bricks. Density of dry bricks of different individual manufacturers follows normal distribution and the coefficient of variation is found to be within 5 percent which again reflects a good

quality control in brick making. Practically all properties such as dimensions, dry density, wet density, percentage of water absorption and compressive strength of bricks are found to follow normal distribution. Cement-sand motar cubes of three different mixes were cast in the laboratory and tested after 28 days to study the variability of mortar strength. Histograms and cumulative distributions of compressive strength of mortar for three different mixes are presented. Normal distribution appears to be a good model to represent the compressive strength of mortar. The statistical variation of joint thickness of mortar in different existing buildings was also studied and found to follow normal distribution at 1 percent significance level.

The strength of brick masonry depends on the quality of brick, mortar, joint thickness and joint layout. Due to non availability of field data, the strength of masonry was generated by Monte Carlo simulation on a digital computer from the randomly generated data of brick and mortar strengths using their statistical properties. Histogram and cumulative distribution of simulated masonry strength are presented. The central region of the simulated data matched with normal distribution but mismatching was observed at either tails. Strength of masonry may still be assumed as normally distributed random variable.

Chi-square test and Kolmogorov-Smirnov test are generally used to examine whether the data of a particular random variable follows an assumed probability distribution. Usual Chi-square test with equal class interval for testing the suitability of a hypothesized probability distribution has limitations. It depends on three arbitrary decisions such as the choice of the starting point, number of classes and width of classes. Thus it may lead to doubtful acceptance or rejection of the null hypothesis. The three arbitrary choices can be made to a single variable if the test is conducted with classes of equal probability. The level of significance at which the null hypothesis can be accepted is found to vary randomly with the choice of number of classes. Therefore, a series of tests should be conducted with different number of classes for a dependable decision. Kolmogorov-Smirnov test (K- S test) does not involve any arbitrary choice of parameters like that of Chi-square test. Standard tables available for use with K-S test are only valid when the parameters of the null distribution is fully specified. Those tables cannot be used when the parameters of the null distribution is not specified but have to be estimated from the data. The critical values of K-S statistic for different level of significance are generated by Monte Carlo simulation and are presented for use with K-S test.

Reliability analysis of reinforced brick beam(RBB) under flexure is presented. Due to random variations in strengths of masonry and steel, there exists a probability that a RBB section will fail as an over-reinforced even though the section is designed as an under-reinforced based on deterministic analysis. A general method of analysis of probability of failure of RBB section is presented treating strengths of masonry and steel as random variables. Probability of failure (p_f) of a RBB section is the sum of the probabilities of the section failing as an under-reinforced (p_{fu}) and failing as an over - reinforced (p_{fo}). The computation of the above probabilities involves evaluation of multiple integrals. As the area of steel in a RBB section is increased, the probability of failing as an under-reinforced (p_{fu}) decreases and the probability of failing as an over-reinforced (p_{fo}) increases but the probability of failure (p_f) decreases for the same external moment. Monte Carlo simulation is also used to study the reliability of the section.

A method of reliability based design of RBB section is presented. The method involves solution of an integral equation to find the area of steel or the dimensions of the section for a preassigned reliability. Examples are given to illustrate the method. Limit state design method based on semi-probabilistic approach is illustrated considering the coefficient of variation of resistance and load. Partial

safety factors applied to strength of materials and load for a target reliability depend on the coefficient of variation of resistance and load. A set of partial safety factors is suggested for limit design of RBB sections.

Aspects of durability of reinforced brickwork are reviewed and discussed. Corrosion of reinforcement in reinforced brickwork appears to be a serious problem. Estimation of time of cracking due to corrosion of reinforcement is formulated. A method of reliability analysis, taking into account the decrease of moment capacity with time, is presented. The load factors or partial safety factors needed for the design of a RBB section for a given design life depends on the rate of deterioration and type of exposure. This is illustrated through an example. Conclusions and recommendations based on the discussions of the results are presented at the end.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Brick masonry construction is one of the oldest and it continues to dominate in residential building construction. Brick units bonded together by mortar in a predetermined orientation used for resisting external loads is called brick masonry. Burnt clay building bricks are produced in India for building construction. The concept of reinforced masonry structural member to resist tensile forces is not new to the Civil Engineering profession. Mild steel flats and rounds have been used for many years in some form or other. The use of high yield strength deformed (HYSD) bars in reinforced brick masonry has become more popular for its improved strength and bond capacity.

Structural safety is provided by means of factor of safety in the conventional working stress design philosophy. The safety of a structural member against external loads is ensured by limiting the actual stresses to a set of permissible stresses whereas the safety is ensured through proper choice of load factors in ultimate strength design. In both these design philosophies load and strength of materials are treated as deterministic. However, the real loads and strength of

materials are probabilistic rather than deterministic. To provide a meaningful safety to a structure, a semi-probabilistic design philosophy using different limit state has come up taking into consideration the variability in material strength and load.

1.2 Literature Review

1.2.1 Brick masonry

The general development of reinforced brick masonry was reviewed by Srivastava (1)* and Dayaratnam et al. (2). Foster (3) has given an extensive review of the development of reinforced and prestressed brick work up to 1981. For the sake of completeness, the main developments are reviewed here. Some of the investigations, even though do not deal with RBM, are reviewed to establish certain relevant strength and stress relations.

It was reported by Beamish (4) and Plummer et al.(5) that Sir M.I. Brunel is the first one to use the concept of reinforced brick masonry in building a chimney in 1813. Results of tests on RBM structural members was reported by Brebner (6). Brebner's work is believed to be the first organised research work carried out in India around 1920. Several tests on Reinforced Masonry beams were carried out by Parsons et al. (7) and Withey (8) during 1932-33. In 1932, the first draft code of the recommended practice and

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* The numerals in parenthesis indicate the reference numbers given at the end.

specification for RBM construction was developed (5). Since then, several investigations were conducted on plain and reinforced brick masonry at different places.

Different codes of practice (9,10,11) outlined the methods of designing plain and reinforced masonry by permissible stress approach. Basic compressive stress of brickwork for different brick strength and different mortar type is given in tabular form. Strength of brickwork in compression can be calculated through basic compressive stress by allowing different reduction factors for slenderness ratio, shape of individual units and eccentricity.

'Building Code Requirements for Reinforced Brick Masonry' (12) published in 1960, and 'Recommended Practice for Engineered Brick Masonry' (13) published in 1969 are the outgrowth of different research works carried out in U.S.A. on plain and reinforced brick masonry. Cutler et al.(14) explained how research informations reported by different countries and individuals have been taken into consideration for developing the Canadian Building Code for masonry. The structural design requirements for load bearing nonreinforced brick masonry walls contained in various standards and codes were compared by Gross et al. (15). Comparison of different standards and codes are difficult because of difference in test methods and requirements of brick and mortar, size and shape of the individual units

as well as the different approaches used to consider the various design factors.

Hallar (16) reported test results of shear test on brick masonry. Several researches were carried out to study the ultimate shear strength of reinforced brickwork beams. Suter and Hendry (17) concluded from experimental results, 'the ultimate shear stress of reinforced brickwork beams increases only slightly with increasing amount of tensile reinforcements but increases significantly with decreasing shear-span to effective depth ratio'. It is recommended that the significant increase in ultimate shear stress with decreasing shear span to effective depth ratio should be recognised in limit state shear provisions. The effect of ratio of shear span to effective depth, the amount of tensile reinforcements and the compressive strength of brickwork on the ultimate shear resistance of RB beams was examined and limit state shear values were proposed (18).

It has been observed that the failure of masonry, loaded in compression is initiated by vertical splitting of bricks. Various attempts have been made to relate the compressive strength of masonry to the tensile strength of bricks (19,20). Stafford Smith (21) discussed existing method of testing brick masonry discs to establish the diagonal tensile strength of brickwork. It was shown through experimental results and finite element analysis that the

diagonal tensile strength of brickwork is approximately equal to either the tensile strength of mortar or the tensile strength of brick, whichever is smaller (21).

Based on the data reported by Talbot and Abrams(22), Parsons(23) developed an empirical formula for estimating the probable compressive strength of concentrically loaded masonry walls or columns constructed with hollow units. The formula relating strength of masonry with strength of brick and mortar can be stated as(23):

$$f_w = k_w \sqrt{(f_b \, f_m)} \qquad (1.1)$$

where f_w = strength of masonry,

f_b = strength of brick,

f_m = strength of mortar,

and k_w = a factor which depends on the orientation of the individual units.

Dayaratnam et al.(2) carried out regression analysis to fit Eq. 1.1 to the experimental results and recommended different range of values for k for calculating the strength of masonry. A formula for predicting strength of masonry from strength of brick and type of mortar is recommended by SCPI(13). Hoath et al.(24) have shown that brickwork cube made with the traditional cement-lime-sand mortar have higher ratio of brickwork cube strength to mortar strength as compared to that of cement-sand mortar. Francis et al.(25)have studied the effect of joint thickness on the

compressive strength of brickwork. Experimental results showed that prism compressive strength decreases as the average joint thickness increases.

The problem of masonry infilled frame was dealt by Carter and Stafford Smith(26). The mode of failure was found to be governed by the geometry of the structure and also by the relative values of the bond shear strength, internal friction and diagonal tensile strength of masonry. Stafford Smith et al.(27) has studied the distribution of stresses in brickwork walls subjected to vertical loads through finite element approach. It was found that the peak tensile stress values in a wall increases with height to length ratio of the wall and with the brick to mortar elastic modular ratio. When a structural frame of building is infilled with masonry, the diaphragm action of the wall in the frame produces substantial increase in the racking stiffness of the structure(26,27). Riddington et al.(28) have shown by finite element approach that the failure loads, the magnitude of diagonal tension and shear stresses are governed almost by the length to height ratio of the infill. The composite action of masonry walls with beams and frames has been studied by Stafford Smith(29). Design methods to take care of the composite action of wall with beams and frames were also discussed in the paper.

The compressive strength of a wall is related to compressive strength of brick and mortar, but it also depends on the height of the bricks used(30). Howard et al.(31) have pointed out the effect of manufacturing variables on the strength and durability of bricks. The effect of construction variables on the strength of brick masonry was also discussed. Grimm and Halsell(32) illustrated the difference between good and bad workmanship, and discussed quality control in brick masonry construction as practiced in U.S.A. Thomas(33) discussed about the quality control and suggested regular testing of bricks and brickwork test cubes. Variables affecting strength of brick masonry was reviewed and summarized by Grimm(34). It was concluded that compressive strength of brick masonry prism is a function of compressive strengths of brick and mortar, workmanship quality, magnitude and direction of load, eccentricity and prism geometry, age and curing. Empirical equations relating strength of brick masonry with different variables are derived based on different research data. Based on the data obtained by different researchers, Plummer(35) commented, 'The wide variations between individual specimens of the same group frequently encountered in such data, are due to large number of variables, some of which are very difficult to control, to which all tests, which might be expected to produce results comparable to those obtained in masonry construction, are

sensible. These variables include air content and flow of mortar, elapsed time between spreading mortar and placing brick in contact with it, suction of brick, pressure or tapping applied to joint during forming, texture of brick surface and probably, to a minor degree at least, other factors which have not been identified. Hendry(36) critically discussed the strength and properties of load bearing brickwork materials with particular reference to workmanship.

Based on statistical properties of masonry, Macchi(37) discussed about the choice of suitable factors of safety which also take into account the structural behaviour. Hendry(38) pointed out, 'Research work which results in the reduction of the coefficient of variation by even a few percent is likely to be of great practical significance in terms of reducing the safety factor and thus achieving efficient use of materials'. Graphs of central safety factor needed for different variability of material strength when designed for a particular probability of failure were given (37,38).

With a view to incorporate separate safety provisions to loads and strength of materials as a more rational approach to the design problem, the limit state format has now been adopted in different national codes (39,40,41) for design of reinforced concrete. Czechoslovakia has already developed their brickwork code in limit state format (42).
After the adoption of limit state format of CP 110: 1972(39), the brick masonry code has been revised and

developed in the same line of reinforced concrete, and as a result BS 5628: Part 1(43) for plain brick masonry is published. At the same time, a committee of engineers and architects produced SP 91 (44) to serve as a guide to the design of reinforced and prestressed masonry.

Stress-strain curves of brick masonry for different cement-sand mortar are presented by Jain et al. (45). Based on experimental investigation, stress - strain relation of brick masonry prisms is derived and presented by Dayaratnam et al. (2). Ultimate load theory similar to reinforced concrete is found to be satisfactory in determining moment capacity of RB beams and slabs (1,2,45). Beard (46) suggested a formula for predicting ultimate strength of reinforced brick beam in flexure using a parabolic stress strain relationship with a falling branch and rejected the enhancement factor included in SP 91 (44). Sinha (47) adopted another approach of predicting the ultimate strength using a curvilinear and cubic parabolic stress block. Recently, British Standard Institution (BSI) has published a draft code BS 5628 : Part 2(48) for reinforced and prestressed brick masonry in a limit state format. A simplified rectangular stress block is adopted in the draft code. Partial safety factors 2.5 and 2.8 for brickwork given in the draft code was criticized by Beard(49). Various clauses in the draft code(48) relating to the shear strength of reinforced brickwork

are discussed by Hendry(50).

1.2.2 Probabilistic concept of safety

The basic concept of structural safety analysis was first discussed by Freudenthal(51) and later by Asplund(52) and Pugsley(53). Freudenthal(51) said, 'With increasing perfection of design methods, the element of 'ignorance' can be largely eliminated; but the element of 'uncertainity' is caused by circumstances that can be changed, to a certain extent, but can never be removed. Hence, the safety factor is a measure of uncertainity rather than ignorance' . The statistical frequency distribution of load and resistance must be considered in determining the reliability. Freudenthal(54) commented, ''Because the design of a structure embodies uncertain predictions of the structural materials as well as the expected load patterns and intensities, the concept of probability must form an integral part of any rational design; any conceivable condition is necessarily associated with a numerical measure of the probability of its occurrence. It is by this measure alone that the structural significance of a specified condition can be evaluated''.

The concept of unserviceability and failure was defined and procedures for computation of probability of failure were outlined by Freudenthal(54). Statistical data

on intermediate grade reinforcing steel and ultimate strength of concrete was presented by Julian(55). Graphs of minimum required factor of safety versus probability of failure for normal and lognormal distributions for load and resistance were shown(55). Methods of providing engineering safety and additional safety necessary for social purposes were formulated by Brown(56). Attempts for providing safety can be made by specifying (i) a minimum ratio of resistance to maximum design load, (ii) a maximum value of probability of failure and (iii) a minimum factor of safety coupled with a maximum probability of failure(56). The concept of structural life along with functional and collapse failures was also discussed.

Methods of safety analysis based on lognormal and extremal distributions of resistance and of load were developed and the effects of redundancy on the risk of failure and reliability function were examined(57). The work of the Task Committee on factors of safety, ASCE, were summarized by Freudenthal, Garelts and Shinozuka(58). Probability of failure (p_f) has been recognised as the probability of the occurence of the event $(R-S) < 0$ or $(R/S) < 1$ and can be computed as (for R and S independent)

$$p_f = P(R < S) = \iint_D f_R(r)\, f_S(s)\, dr\, ds \qquad (1.2)$$

in which D is the region of integration defined by the event $(R < S)$, $f_R(r)$ and $f_S(s)$ are the probability density functions

of resistance R and load S respectively. Reliability functions were derived for the cases when there is N repeated load application at equal intervals or at prescribed instance and the number of occurence of load is governed by the Poisson's law. Numerical examples of trussed tower and aircraft against wind load were given to illustrate the method. Derivation of reliability functions was also extended for structures with multiple members. Bounds on reliability of structural system were presented by Cornell(59). Reliability theory of deteriorating structures was formulated by Kameda and Koike(60) considering resistance deterioration under repeated random loads.

An extensive review of the available literature till December, 1971 on structural safety was presented by the Task Committee on Structural Safety(61). The choice of safety level in structural design is one of the most difficult job. The choice of safety level based on minimum expected loss criterion was formulated by Turkstra(62). Ang and Amin(63) formulated a method for the determination of reliability of a statically determinate system and established a monotonically decreasing property for the hazard function. The method was also extended for indeterminate system and numerical examples were given. Ang and Amin(64) proposed a new concept of structural safety where the lack of knowledge and information was distinguished from statistical

dispersions. The lack of knowledge and information was handled through a judgement factor whereas the statistical variables were treated probabilistically. Reasons and advantages of probabilistic studies of safety were discussed and the basic probabilistic procedures in the analysis of structural safety were introduced by Shah(65) using only mean values and standard deviations of resistance and load for an assumed probability distribution of (R-S) or (R/S).

Sexsmith and Nelson (66) discussed several limitations in the application of probabilistic concepts. The major difficulties were recognised as (i) the choice of the appropriate probability model, (ii) introduction of subjective elements into the probabilistic structure and (iii) interpretation of probabilistic information in a form which leads to rational decisions. Different techniques to avoid these difficulties were presented by others(67,68). The approximate probability measures associated with the existing code provisions was investigated by Benjamin and Lind(67) and reliability based procedures were illustrated using a set of assigned probabilities. The concept given in the paper is to relate probability measures to existing deterministic code provisions thereby allowing improvement of design procedures within the existing deterministic code format. It was shown that the design is relatively insensitive to these probabilities so that a code making body has

considerable freedom in assigning such measures of probabilities. A consistent first order code format based on first and second moments of all stochastic variables was presented by Cornell(68). This approach was extended to code formats of arbitrary higher order than first and second moments of stochastic variables by Lind(69) and the calibration of a partial safety factor format to Cornell's(68) format as well as to Ang-Amin(64) format was demonstrated.

Systematic analysis of uncertainity through first order statistical analysis and the explicit use of probability of failure or survival as a measure of risk and safety were suggested by Ang(70). Practical methods of risk assessment in terms of failure probability and for the development of reliability based design criteria were described and developed. Hasofer and Lind(71) presented an exact and invariant code format and described a natural measure of the second-moment reliability index of a design as the distance from the mean of basic variable vector to the failure region boundary when all the variables are measured in standard deviation units. It was shown that this measure does not depend in any way on the precise analytical form of the failure criterion.

Systematic evaluation of risk and analysis of uncertainity were developed by Ang and Cornell(72). Under conditions of uncertainity, safety and serviceability of

structures can be assured only in terms of the probability of survival (or conversely of failure). Uncertainity (including uncertainity associated with errors in estimation and imperfection in mathematical models) was expressed in terms of the coefficient of variation. Formulation for multiple load cases was also presented. It was emphasized that the distribution of safety margin (R-S) or ln(R/S) is important in the calculation of probability of failure and not the distribution of the individual variates(70,72). At high risk levels, e.g., $p_f \geq 10^{-3}$, the calculated failure probability p_f is not very much different regardless of the type of the distribution,however it depends significantly on the choice of the distribution of (R-S) or ln(R/S) for a low risk level ($p_f \leq 10^{-5}$). It was argued (70,72) that the relative measure of risk is all that is necessary and for this purpose a prescribed distribution of (R-S) or ln(R/S) should suffice. A distribution function,composed of a central gaussian portion and an exponential tail with a single point of discontinuity, was proposed by Lind(73) as a convenient representation of statistical demand and capacity data to evaluate the reliability, regardless of any conjecture about the parent distribution. The convolution integral of the probability of failure was evaluated in closed form using simple statistics of the data. Ellingwood and Ang(74) formulated a risk based design criteria and showed how the

level of risk is affected by the uncertainities of the design variables. The analysis of design uncertainities arising from the statistical nature of some variables and incomplete information regarding others was examined. Ravindra et al.(75) illustrated the design method proposed by Ang and Cornell(72) with practical examples and showed that the code parameters such as safety indices can be selected to match the safety level of current designs. Only mean values and standard deviations were used to represent the design random variables and the final design criteria were developed on a probability-distribution free basis.

Models of structural reliability of systems including determinate structures (weakest link system), indeterminate structures(fail safe system) and brittle members were described by Moses(76). Using the design format suggested by Ang and Cornell(72), Moses (76) introduced partial safety factor defined as the quantity by which the element safety factor should be changed to account for its presence in a structural assemblage so that the overall failure probability remains roughly equal to what is desired for the element.

Costello and Chu(77) presented a formulation for calculation of failure probabilities of reinforced concrete rectangular beams with and without compression reinforcements assuming beta distribution for strengths of concrete and

steel. Warner and Kabaila(78) described a Monte Carlo simulation technique to determine the cumulative distribution functions of stochastic variables which are functions of several stochastic variables. The cumulative distribution function of ultimate strength of an axially loaded short reinforced concrete column was investigated using this technique and the results were compared with closed form solution. Procedures for determining the probability distributions of the safety margin and related quantities of several types of reinforced concrete members were presented by Sexsmith (79). Using Monte Carlo technique, Allen (80) made a detailed study on estimated probability distributions for the ultimate bending strength and ductility ratio (ratio of curvature at ultimate to curvature at yield) of rectangular concrete sections reinforced in tension only. It was concluded that there is a significant probability that an under-reinforced section designed according to ACI 318-63(81) would undergo a brittle compressive failure. The variability of ductility ratio was found much higher than that of ultimate bending strength. Reliability analysis of prestressed concrete simply supported beams designed on the basis of Indian Standards, American Concrete Institute and Recommended Code of Practice by CEB-FIP(82) was presented by Chandrasekar and Dayaratnam (83, 84). Ranganathan and Dayaratnam (85,86)used

Monte Carlo Simulation technique to evaluate the probability of failure of a prestressed concrete flanged section. The failure of the section at limit state of strength was divided into two cases :(i) under-reinforced and (ii) over-reinforced and the probability of failure was evaluated based on the above two conditional failure events.

Moses and Kinser (87) discussed on optimum sizing of elements for multielement and multiload conditions with safety defined in terms of allowable probability of failure. Some examples (trusses and frames) of the optimal reliability based design were presented by Moses and Stevenson (88). Rao (89) demonstrated the feasibility of optimum cost design of under-reinforced concrete beams with a reliability based constraint. The design was considered to be safe when the expected ultimate moment capacity exceeds the design moment by a certain number of standard deviations. Optimization of prestressed concrete beams subject to reliability constraint in addition to the normal constraints was presented by Chandrasekar and Dayaratnam(90).

1.3 Statement of the Problem

The design of reinforced brick masonry in India is being carried out by working stress method. The safety of the structure is ensured by limiting the actual stresses less than or equal to permissible stresses. A semi-probabilistic or probabilistic design method which

incorporates the uncertainity in strengths of materials and load is more rational. A semi-probabilistic or probabilistic design criteria similar to that of reinforced concrete is to be developed to provide a rational safety margin in the design of reinforced brick masonry. The object of the present investigation is as follows:

(a) Statistical analysis of physical and some important mechanical properties of brick and mortar.

(b) Prediction of empirical probability laws of strengths of brick, mortar and masonry.

(c) Presentation of a method of reliability analysis of reinforced brick beam in flexure.

(d) Design of reinforced brick beams for a given probability of failure.

(e) Study of the variation of load factors with coefficient of variation of resistance of the section and probability of failure.

(f) Durability study and analysis of probability of life expectancy of reinforced brick beams.

The organization of the thesis is described in the following paragraphs. The general introduction of reinforced brickwork along with the relevant literatures on the general development of reinforced brickwork and probabilistic concept of safety are presented in Chapter 1.

Chapter 2 deals with the statistical analysis of field da on brick and mortar. Dimensional and strength properties of bricks were measured in the laboratory and statistical analysis was carried out. Cement-sand mortar cubes of three different mixes were cast in the laboratory and tested after 28 days of curing. Statistical analysis of strength of mortar was also carried out to study the variability of mortar strength. Due to nonavailability of field data, strength of masonry was generated by Monte Carlo simulation using the deterministic formula relating the strengths of brick, mortar and masonry. Samples of masonry strength were generated from the specified distribution of strengths of brick and mortar and the statistical analysis was carried out. Chi-square test was conducted to test the suitability of normal and lognormal distributions to the data of different dimensional and strength properties. Finally, relation between characteristic and mean strength of masonry was established assuming normal and lognormal model for masonry strength.

Reliability of chi-square test is discussed in Chapter 3. Usual chi-square test depends on three arbitrary decisions such as the choice of the starting point, number of classes and width of classes selected for the test and thus may lead to doubtful results. Some of these problems are illustrated through examples and a practical procedure

is suggested for a dependable decision. Kolmogorov-Smirnov test (K-S test) does not involve any arbitrary choice of parameters like that of chi-square test. Standard tables available for use with the Kolmogorov-Smirnov test are only valid when the null distribution is completely specified. The critical values of K-S statistic for different level of significance are generated by Monte Carlo Simulation for use with Kolmogorov-Smirnov test when the parameters of the null distribution is not specified but have to be estimated from the data.

Chapter 4 of the thesis deals with the reliability analysis of reinforced brick beam (RBB) at flexural strength. The failure of a reinforced brick beam section is divided into two cases (i) under-reinforced and (ii)over-reinforced. A general method of analysis of probability of failure of RBB section is presented based on the above two failure events. Monte Carlo simulation is also used to study the reliability of the section and a comparative study is presented.

A method of reliability based design of RBB section is presented in Chapter 5. The method involves solution of an integral equation to find the area of steel or dimensional properties of the section. Examples are given to illustrate the method. Design method based on semi-probabilistic approach is illustrated considering the coefficientsof variation of resistance and load.

Aspects of durability of reinforced brickwork are reviewed and discussed in Chapter 6 with particular reference to corrosion of reinforcement. Estimation of time to cracking due to corrosion of reinforcement is formulated. A method of reliability analysis and design considering strength deterioration of RB beam is also presented. Conclusions and recommendations drawn based on the discussions of the results are presented at the end.

CHAPTER 2

STATISTICAL ANALYSIS OF PROPERTIES OF BRICK, MORTAR AND MASONRY

2.1 Introduction

Brick and brick construction has been used in building activities for centuries and it continues to dominate among the commonly used building materials. Reinforced brick masonry construction has got popularity in this century because of its low cost compared to reinforced concrete. The design of reinforced brick masonry requires a knowledge of the properties of brick, mortar and masonry strength. The safety against load is ensured and reflected by permissible stresses in working stress design and by use of load factors and partial safety factors in ultimate strength design or limit state design philosophy. In the deterministic methods of design, the design variables are treated as deterministic even though there is a certain degree of variability in them. On the other hand, the semiprobabilistic methods of designs incorporate the statistical variations in some of the design variables. The statistical variations in brickwork depend very much on the constructional practice and the degree of quality control.

The production of brick and brick masonry construction in India is highly labour intensive. The production of bricks in India is rural oriented by and large, and it appears to use hand moulding methods which have been in practice for centuries. Statistical analysis of different parameters such as dimensional and strength properties of brick and brickwork is presented in this chapter.

2.2 Statistical Analysis of Bricks

2.2.1 Sample collection and notations

For statistical analysis of brick properties, about 100 samples of bricks were collected from each of eight different manufacturers. Three manufacturers were selected from Kanpur zone and the rest five were from Faroke zone in Kerala State. The set of bricks from a particular manufacturer collected at a time will be called as a fixed lot in subsequent discussions. About 100 samples were selected at random from each manufacturer as the representative of different fixed lots and corresponding notations used in the subsequent sections are as follows:

Zone	Notation*	Remarks
Kanpur	BK1	Fixed Lot
Kanpur	BK2	
Kanpur	BK3	
Faroke (Kerala)	BK4	
Faroke (Kerala)	BK5	
Faroke (Kerala)	BK6	
Faroke (Kerala)	BK7	
Faroke (Kerala)	BK8	

* Name of the manufacturer is not indicated here.

A mixed lot is a combination of different fixed lots. Data of fixed lots of bricks of Kanpur zone was combined and called mixed lot 1. Similarly data of Faroke zone was also combined as mixed lot 2 and the statistical analysis is carried out.

The Structural Engineering Laboratory of I.I.T. Kanpur has been associated with the testing of bricks supplied by different manufacturers. Brick samples of about 6 bricks each called as set were supplied by different manufacturers at different time to the laboratory for testing. The data of those test results were taken for statistical analysis and will be called a random lot. The random lot consists of 19 sets (total of 113 samples)

and can be considered as the representative sample of random collection of bricks from different manufacturers. The summary of mixed lot and random lot is given below:

Lot type	Zone	Notation	Remarks
Combination of BK1, BK2 and BK3	Kanpur	BK9	Mixed lot 1
Combination of BK4, BK5, BK6, BK7 and BK8	Faroke	BK10	Mixed lot 2
Random collection	Uttar Pradesh	BK11	Random lot

2.2.2 Parameters of interest

The parameters which were measured directly from the brick specimens are called measured parameters. The following parameters were chosen to study their statistical variations:

i) Length of brick = L (in mm)

ii) Breadth of brick = B (in mm)

iii) Height of brick = H (in mm)

iv) Dry weight of brick = W_d (in kg.)

v) Saturated weight of brick = W_s (in kg.)

vi) Compressive Load = P (in kg.)

From the above measured parameters, the following parameters were computed:

i)	Area (flatwise)	= A (in cm^2)
ii)	Volume	= V (in cm^3)
iii)	Dry density	= γ_d (in kg/m^3)
iv)	Wet or saturated density	= γ_w (in kg/m^3)
v)	Percent of water absorption	= p (percent)
vi)	Compressive strength of brick (flatwise)	= f_b (in N/mm^2)

The measuring and computing procedures of different parameters are discussed briefly in the following section.

2.2.3 Measuring procedures

Length, breadth and height of a brick specimen were measured upto 1 mm accuracy by slide calipers. Each dimension was measured at four different sections and the mean of the four was taken as the value of the parameter for that specimen. Each brick specimen was dried in a ventilated oven at a temperature of 110°C for 24 hours and then cooled to room temperature. Dry weight (W_d) was taken. The specimen was then immersed completely in clean water at room temperature (27 ± 2°C) for 24 hours to saturate the specimen. After 24 hours, the specimen was removed and all excess surface water was wiped out by a damp cloth. The wet weight (W_s) was taken and the percent of water absorption

was calculated by the following formula (91)

$$p \text{ (percent)} = \frac{W_s - W_d}{W_d} \times 100 \qquad (2.1)$$

Compressive strength test was done according to the specification given in Indian Standard Code (92). Frogs and all voids were filled with 1:1 cement sand mortar and the sample was cured under damp jute bags for 24 hours and then under water for the following 3 days. After 3 days immersion in water, the specimen was removed and traces of moisture was wiped out. The specimen was then placed on a compression testing machine with flat faces horizontal and mortar filled face upwards. Maximum compressive load (P) was recorded at which the specimen failed. Flatwise compressive strength is then calculated as

$$f_b = \frac{P}{A}$$

where A = flatwise cross-sectional area of the specimen

= L B.

2.2.4 Statistical analysis and histograms

Mean and standard deviation are the two most important statistics of a random variable. If $X_1, X_2, \ldots X_n$ are the n different observations of a random variable X, then

$$\text{Mean } X_m = \frac{1}{n} \sum_{i=1}^{n} X_i \qquad (2.2)$$

$$\text{(S.D.) } s_X = \left[\sum_{i=1}^{n} \frac{(X_i - X_m)^2}{n-1} \right]^{1/2} \qquad (2.3)$$

The above formula gives an unbiased estimate of standard deviation and the co-efficient of variation (δ_X) is given by

$$(\text{C.O.V.})\ \delta_X = \frac{s_X}{X_m} \tag{2.4}$$

The variability of a random variable is reflected through the coefficient of variation. Skewness and kurtosis, are calculated as

$$g_1 = \frac{1}{n} \sum_{i=1}^{n} (X_i - X_m)^3 \tag{2.5}$$

and $$g_2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - X_m)^4 \tag{2.6}$$

Coefficient of skewness and coefficient of kurtosis are computed as follows

$$C_s = g_1 / s_X^3 \tag{2.7}$$

$$C_k = g_2 / s_X^4 \tag{2.8}$$

where g_1 = skewness of X

g_2 = kurtosis of X

C_s = coefficient of skewness

C_k = coefficient of kurtosis

The approximate frequency distribution of a random variable is represented by histogram. The approximate number of intervals for drawing the Histograms is selected as (93)

$$k = 1 + 3.3 \log_{10}(n) \tag{2.9}$$

where k = the number of class intervals

and n = the total number of samples.

The statistical analysis of length (L), breadth(B), height (H), area (A), volume (V), dry density (γ_d), wet density (γ_w), percent of water absorption (p %) and flatwise compressive strength (f_b) of the brick data for eight different fixed lots are given in Table 2.1 through Table 2.8. The statistical analysis of mixed lots BK9 and BK10 are given in Table 2.9 and Table 2.10 respectively. The statistical analysis of random lot BK11 is given in Table 2.11. Units of the parameters are not indicated in the tables and should be read as given in section 2.2.2.

Histograms and the cumulative probability distribution of L, B, H, A, V, γ_d, γ_w, p (%) and f_b of fixed lots BK1 and BK8 are shown in Fig. 2.1 through Fig. 2.18. The variation of mean value and coefficient of variation of compressive strength of brick of different sets of random lot BK11 is shown in Fig. 2.19. Fig. 2.20 shows the histogram of the random lot BK11. A summary of statistical analysis of two fixed lots, two mixed lots and random lot for parameters L, H, p (%) and f_b is also given in Table 2.12 for ready reference.

2.2.5 Chi-square test

Histogram of the data represents approximate frequency distribution. Several nonparametric statistical tests are

Table 2.1 : Statistical Analysis of Fixed lot BK1

n = 100

Parameter	Mean	δ(%)	C_s	C_k	Range	
					Lower	Upper
L	230.7	1.2	0.061	2.403	224.7	236.5
B	110.8	2.0	-0.127	3.028	104.0	116.0
H	64.5	2.4	-0.283	2.647	60.2	68.0
A	255.6	2.8	0.056	3.216	236.1	273.2
V	1649.8	4.3	-0.201	2.230	1483.8	1784.5
γ_d	1579.2	3.1	0.731	3.546	1485.1	1747.8
γ_w	1790.3	2.4	0.684	4.980	1691.8	1968.7
p	13.41	14.7	-0.146	3.040	8.2	18.43
f_b	23.69	23.2	0.379	2.767	13.92	39.35

Table 2.2 : Statistical Analysis of Fixed lot BK2

n = 95

Parameter	Mean	δ(%)	C_s	C_k	Range	
					Lower	Upper
L	227.5	1.5	-0.596	3.586	215.5	234.8
B	108.1	2.1	-0.298	2.591	102.3	112.5
H	62.1	2.8	0.440	2.641	58.3	66.5
A	245.8	2.9	-0.611	4.067	220.4	260.1
V	1526.7	3.9	0.077	3.080	1357.6	1681.9
γ_d	1717.1	4.9	0.782	4.002	1558.9	2023.4
γ_w	1947.4	3.9	0.359	3.469	1777.7	2202.4
p	13.47	14.2	-0.715	4.277	7.53	18.19
f_b	21.49	24.6	0.809	4.141	8.26	39.69

Table 2.3 : Statistical Analysis of Fixed lot BK3
n = 100

Parameter	Mean	δ(%)	C_s	C_k	Range	
					Lower	Upper
L	227.3	1.5	-0.309	2.226	219.8	234.3
B	109.4	1.9	-0.749	4.121	101.8	114.0
H	63.3	3.4	0.047	3.041	57.3	68.8
A	248.7	3.0	-0.546	3.053	226.1	263.3
V	1573.6	5.3	-0.139	2.723	1373.0	1776.4
γ_d	1557.9	5.0	0.163	2.535	1398.3	1768.7
γ_w	1841.1	3.2	0.596	3.851	1719.8	2039.6
p	18.31	17.7	0.268	1.975	13.58	26.76
f_b	14.57	22.5	0.795	3.629	8.05	25.58

Table 2.4 : Statistical Analysis of Fixed lot BK4
n = 100

Parameter	Mean	δ (%)	C_s	C_k	Range	
					Lower	Upper
L	235.8	0.3	0.424	2.637	234.0	238.0
B	116.3	0.6	-0.030	3.556	114.2	117.9
H	73.6	1.6	0.097	2.150	71.4	75.9
A	274.1	0.7	0.006	2.941	268.4	278.5
V	2016.6	1.8	0.245	2.622	1945.7	2101.9
γ_d	1611.1	1.7	-0.619	3.897	1509.8	1668.2
γ_w	1899.6	1.6	-0.522	4.671	1779.2	1976.6
p	17.92	6.1	0.179	6.534	14.13	22.22
f_b	7.60	16.9	-0.437	3.301	3.84	10.99

Table 2.5 : Statistical Analysis of Fixed lot BK5
n = 97

Parameter	Mean	δ (%)	C_s	C_k	Range	
					Lower	Upper
L	231.4	0.7	0.888	2.728	230.0	235.0
B	119.2	1.4	-0.998	7.915	110.7	123.5
H	79.7	1.8	-2.513	14.926	71.2	82.4
A	275.8	1.9	0.270	3.238	260.1	290.2
V	2197.3	2.5	0.352	4.482	2012.3	2368.2
γ_d	1174.0	2.2	1.976	10.815	1700.1	1982.8
γ_w	2036.0	2.1	1.913	11.279	1927.4	2261.1
p	14.78	5.1	-0.399	4.546	12.05	17.05
f_b	14.16	16.3	0.187	2.097	9.52	19.53

Table 2.6 : Statistical Analysis of Fixed lot BK6
n = 100

Parameter	Mean	δ (%)	C_s	C_k	Range	
					Lower	Upper
L	231.0	0.5	0.645	3.023	229.0	234.0
B	117.8	0.7	0.576	3.331	116.0	120.5
H	78.8	1.2	0.307	2.665	77.0	81.3
A	272.2	1.1	0.651	3.462	266.3	281.0
V	2146.0	1.8	0.180	2.996	2064.0	2251.0
γ_d	1795.1	1.3	-0.253	2.859	1724.4	1846.6
γ_w	2038.6	1.1	-0.126	2.419	1976.6	2082.1
p	13.57	4.7	-1.188	6.123	10.71	14.63
f_b	10.85	15.1	0.202	2.421	7.50	14.39

Table 2.7 : Statistical Analysis of Fixed lot BK7
n = 100

Parameter	Mean	δ (%)	C_s	C_k	Range	
					Lower	Upper
L	232.7	0.9	-0.081	3.070	227.0	238.0
B	113.2	1.4	-0.372	2.946	108.9	117.0
H	70.5	2.1	0.277	3.224	67.5	74.8
A	263.4	2.0	-0.224	3.101	249.7	275.0
V	1857.9	2.9	0.060	3.022	1722.9	2003.5
γ_d	1606.0	2.0	0.334	3.520	1518.4	1693.3
γ_w	1885.1	1.8	-0.115	3.077	1788.3	1973.4
p	17.39	5.1	-2.411	11.928	12.68	18.94
f_b	8.13	18.6	0.754	3.130	5.54	12.89

Table 2.8 : Statistical Analysis of Fixed lot BK8
n = 100

Parameter	Mean	δ (%)	C_s	C_k	Range	
					Lower	Upper
L	232.8	0.7	0.170	2.031	230.0	236.0
B	113.8	1.3	0.681	3.141	111.0	119.0
H	76.1	1.6	0.168	2.193	73.5	78.5
A	265.0	1.8	0.602	2.482	257.6	279.7
V	2015.8	2.4	0.566	3.075	1931.4	2153.3
γ_d	1607.6	1.6	-0.174	2.987	1532.5	1659.9
γ_w	1904.4	1.5	-0.243	4.196	1797.2	1981.3
p	18.47	6.3	3.420	21.262	16.12	25.16
f_b	8.58	17.6	0.324	2.582	5.54	12.40

Table 2.9 : Statistical Analysis of Mixed lot 1(BK9)
n = 295

Parameter	Mean	δ(%)	C_s	C_k	Range	
					Lower	Upper
L	228.5	1.6	-0.423	3.121	215.5	236.5
B	109.4	2.3	-0.265	3.217	101.8	116.0
H	63.3	3.3	-0.045	2.483	57.3	68.8
A	250.1	3.3	-0.204	3.504	220.4	273.2
V	1584.3	5.5	0.062	2.446	1357.6	1784.5
γ_d	1616.4	6.2	0.666	3.705	1398.3	2023.4
γ_w	1858.1	4.8	0.768	3.255	1691.8	2202.4
p	15.09	22.3	0.820	3.583	7.53	26.76
f_b	19.89	31.0	0.589	3.005	8.05	39.69

Table 2.10 : Statistical Analysis of Mixed lot 2(BK10)
n = 497

Parameter	Mean	δ(%)	C_s	C_k	Range	
					Lower	Upper
L	232.7	1.0	0.254	2.001	227.0	238.0
B	116.0	2.3	-0.092	2.527	108.9	123.5
H	75.7	4.8	-0.315	1.991	67.5	82.4
A	270.1	2.4	-0.307	3.010	249.7	290.2
V	2045.8	6.2	-0.227	2.389	1722.9	2368.2
γ_d	1678.2	5.5	0.430	1.765	1509.8	1982.8
γ_w	1952.3	3.9	0.395	2.328	1779.2	2261.1
p	16.43	12.9	-0.049	2.776	10.71	25.16
f_b	9.84	29.8	0.885	3.323	3.84	19.53

Table 2.11 : Statistical Analysis of Random lot(BK11)

$n = 113$

Parameter	Mean	δ(%)	C_s	C_k	Range	
					Lower	Upper
L	227.2	2.1	-1.012	9.723	201.0	241.0
B	109.2	2.8	-0.052	2.940	102.0	116.0
W	2815.5	4.8	0.630	2.925	2542.0	3220.0
p	12.05	22.8	0.291	3.299	6.00	19.40
f_b	22.18	29.8	0.451	2.488	11.38	38.85

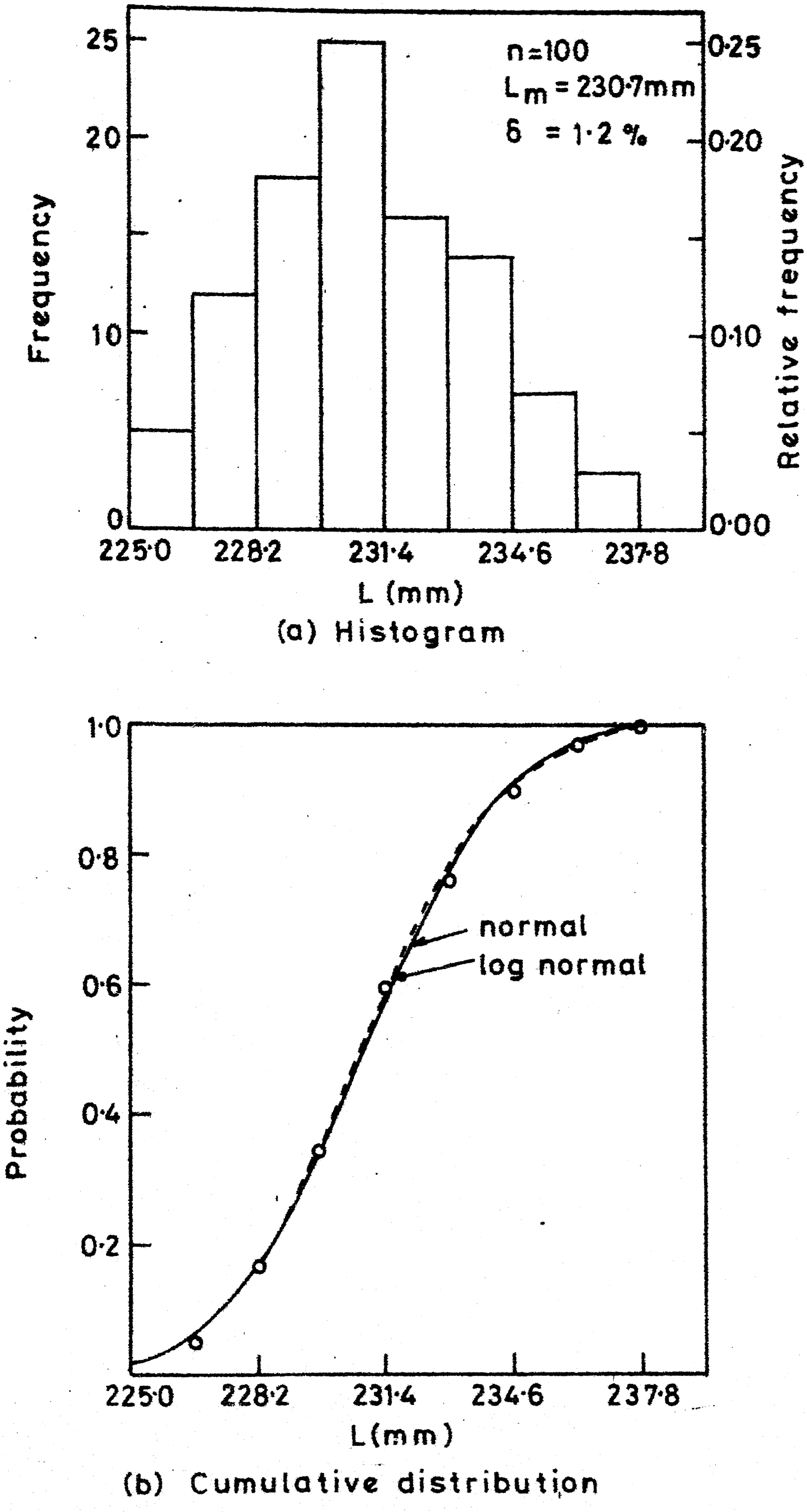
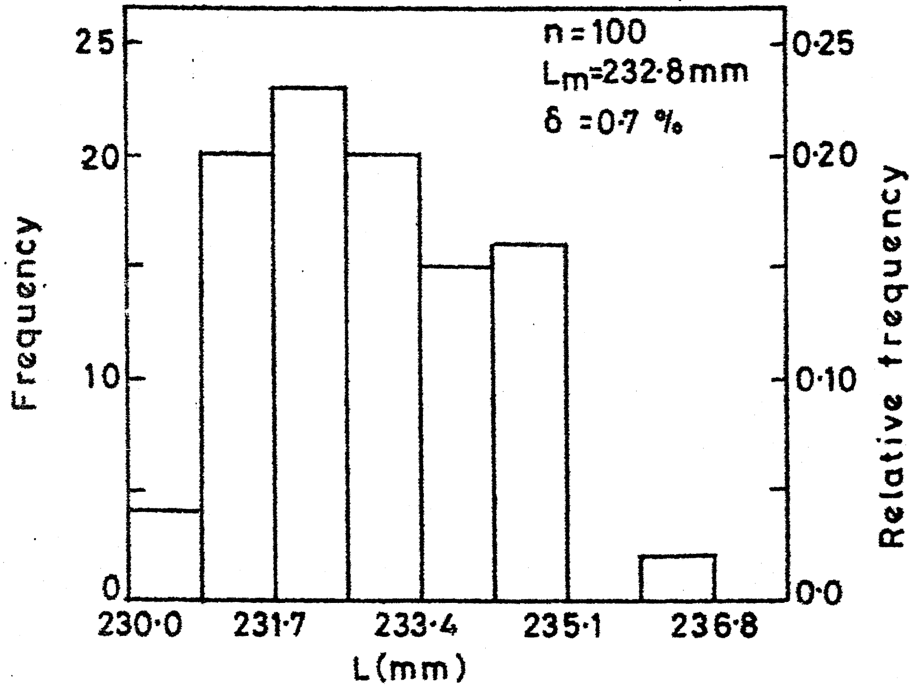
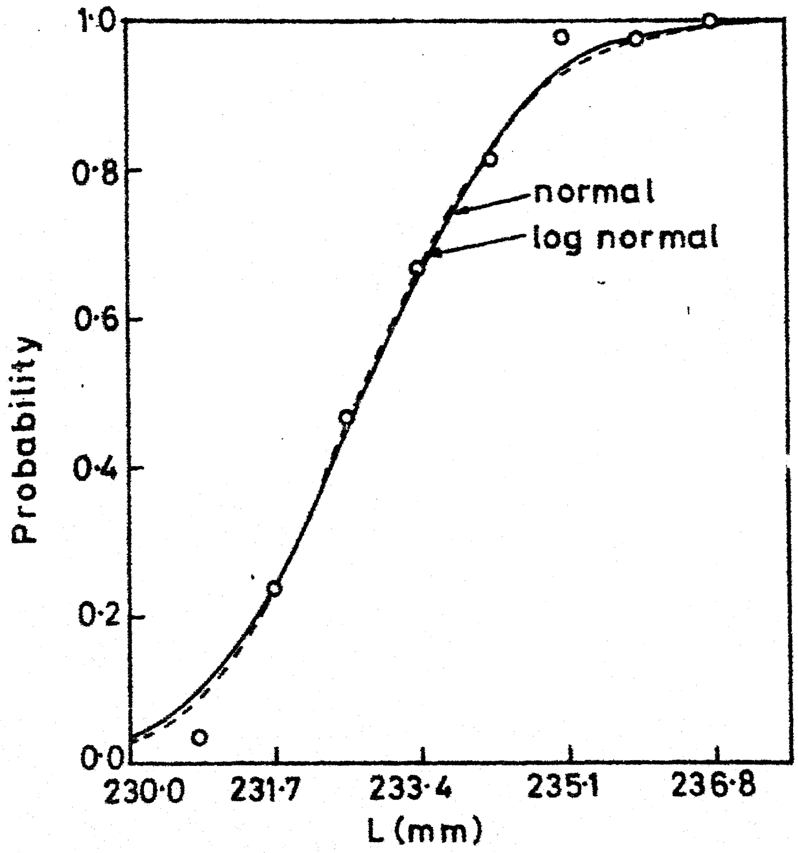


Fig. 2·1 Histogram of length of brick of fixed lot BK1

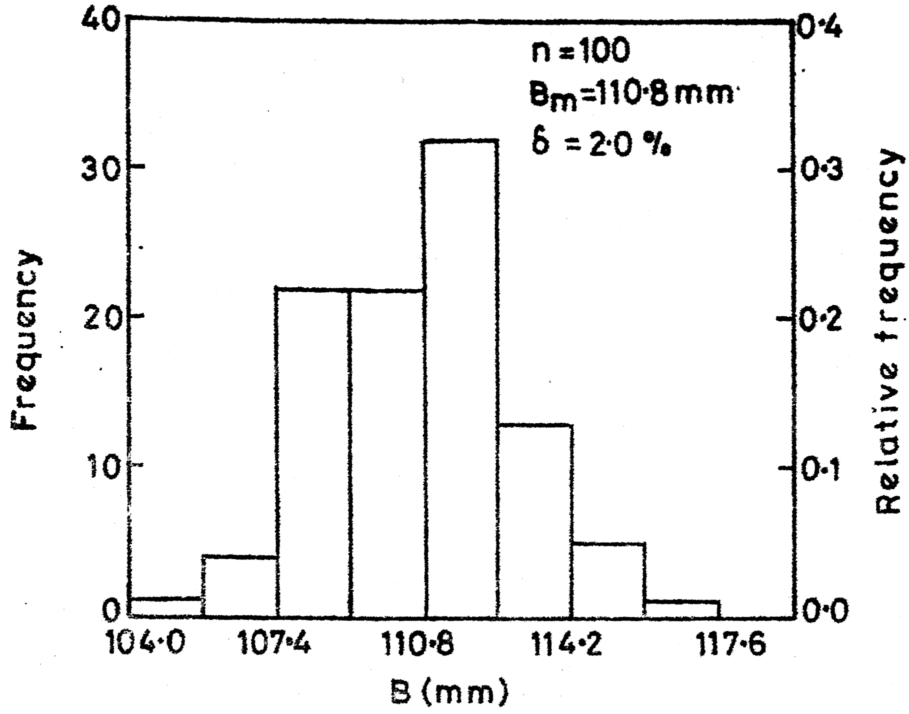


(a) Histogram

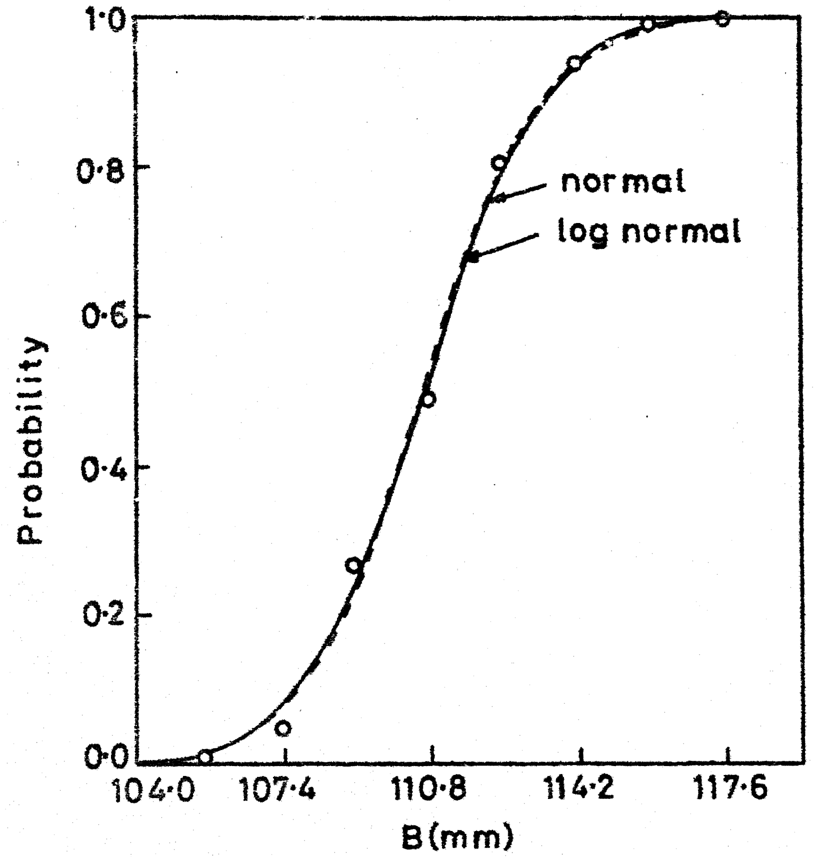


(b) Cumulative distribution

Fig.2·2 Histogram of length of brick of fixed lot BK8



(a) Histogram



(b) Cumulative distribution

Fig.2·3 Histogram of breadth of brick of fixed lot BK1

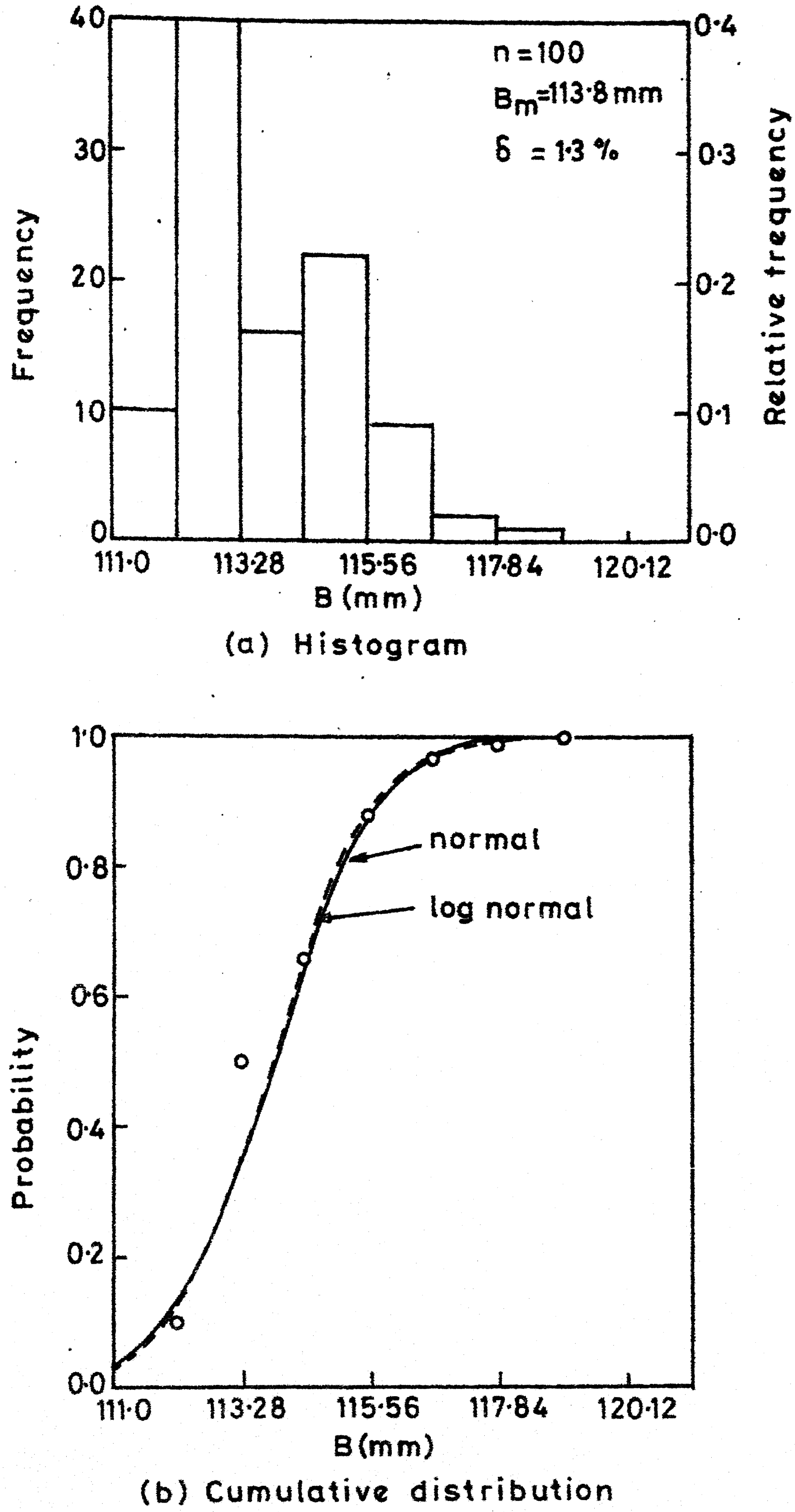
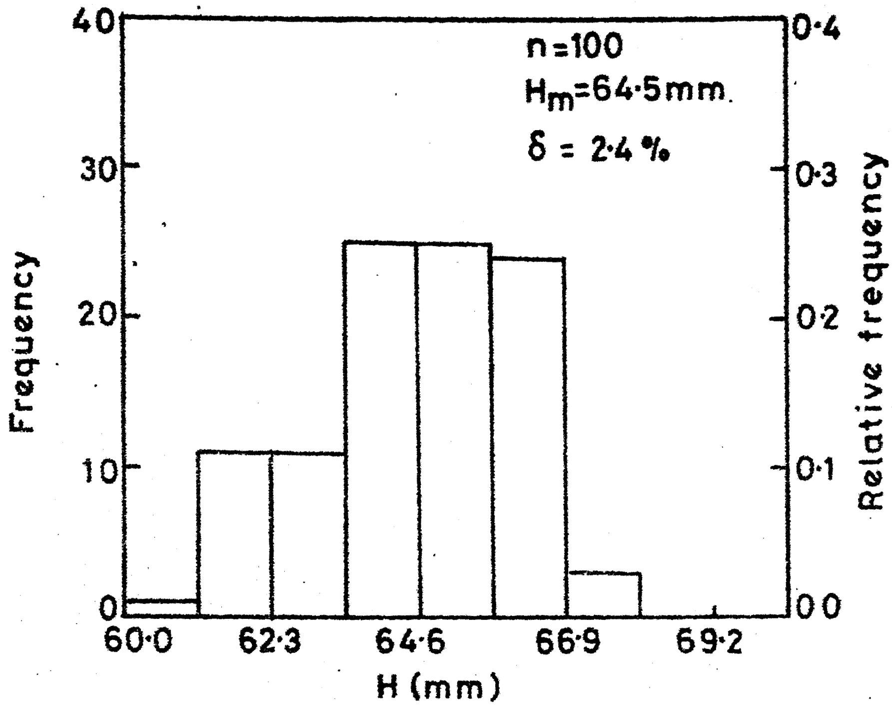
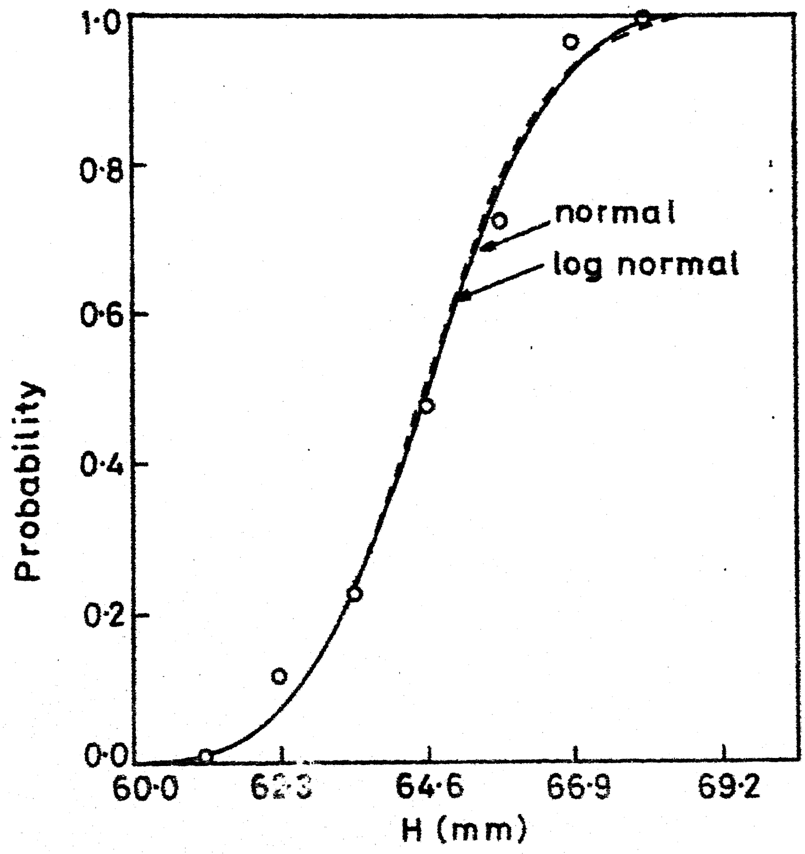


Fig. 2·4 Histogram of breadth of brick of fixed lot BK8

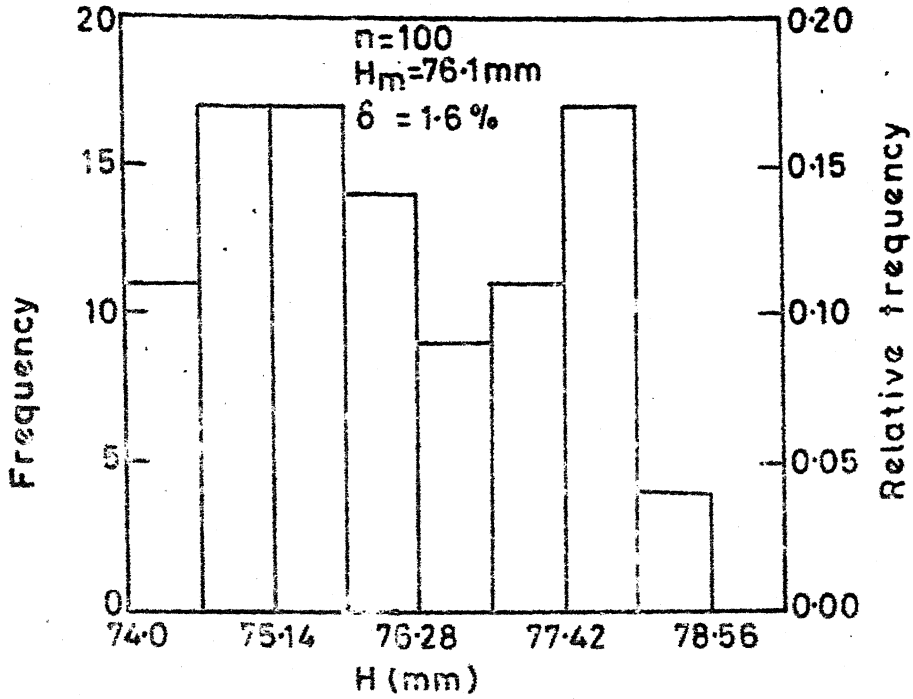


(a) Histogram

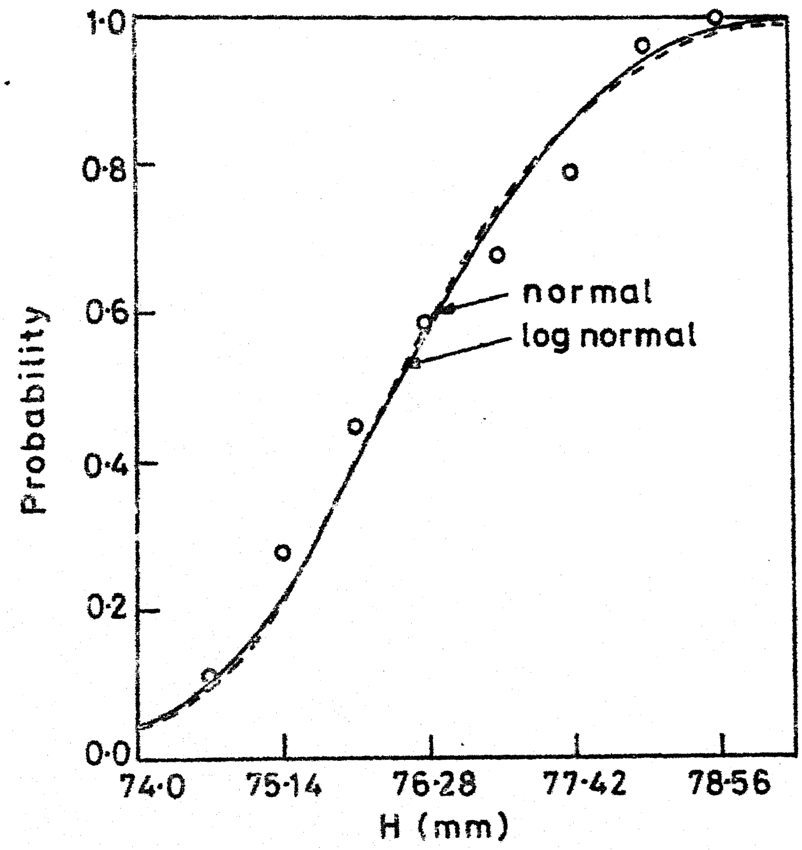


(b) Cumulative distribution

Fig. 2·5 Histogram of height of brick of fixed lot BK1



(a) Histogram



(b) Cumulative distribution

Fig.2·6 Histogram of height of brick of fixed lot BK 8

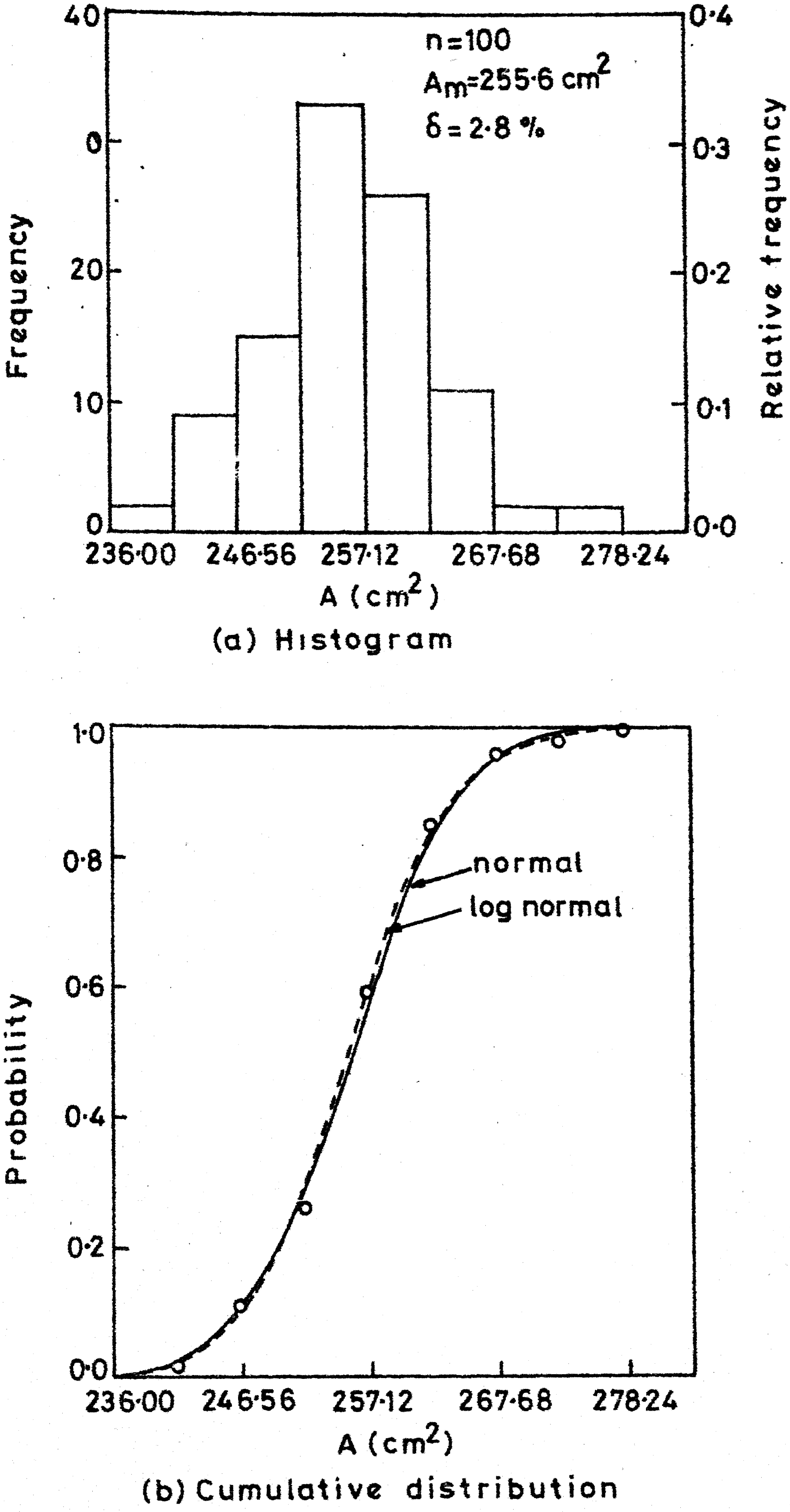
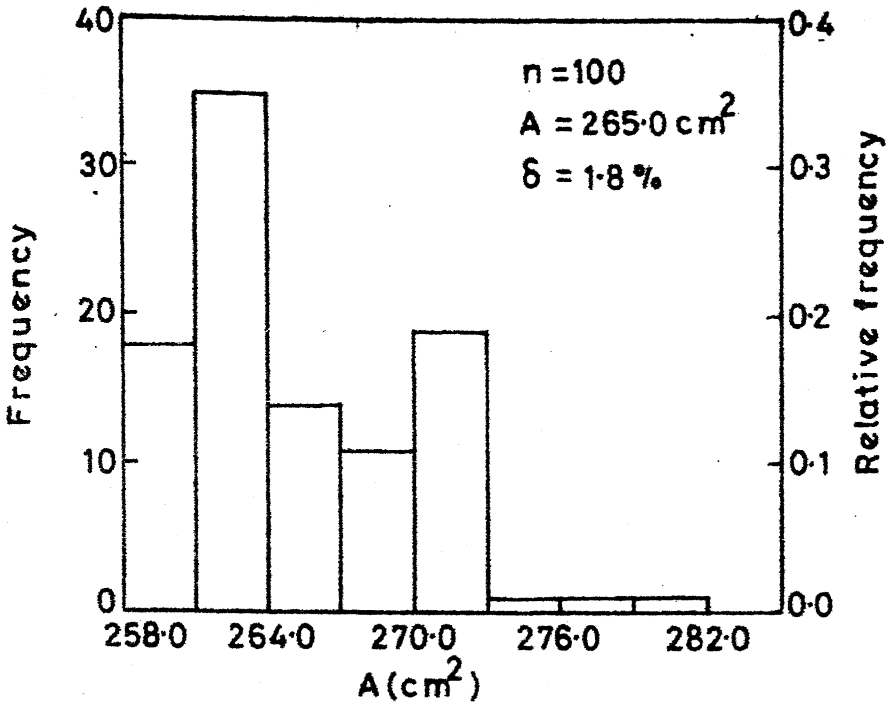
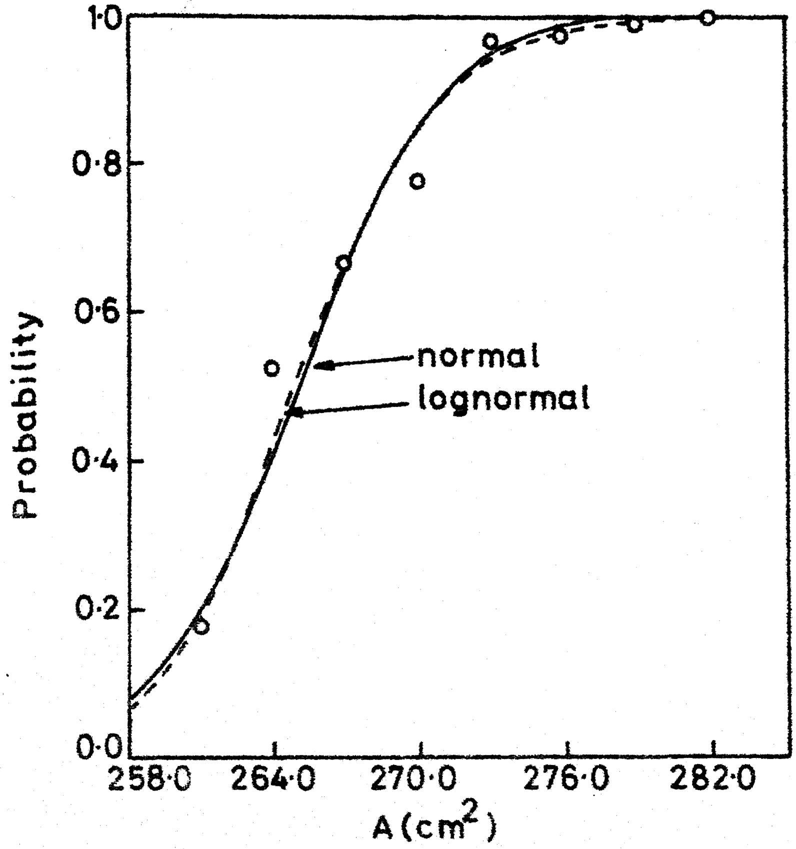


Fig.2·7 Histogram of area of brick of fixed lot BK1

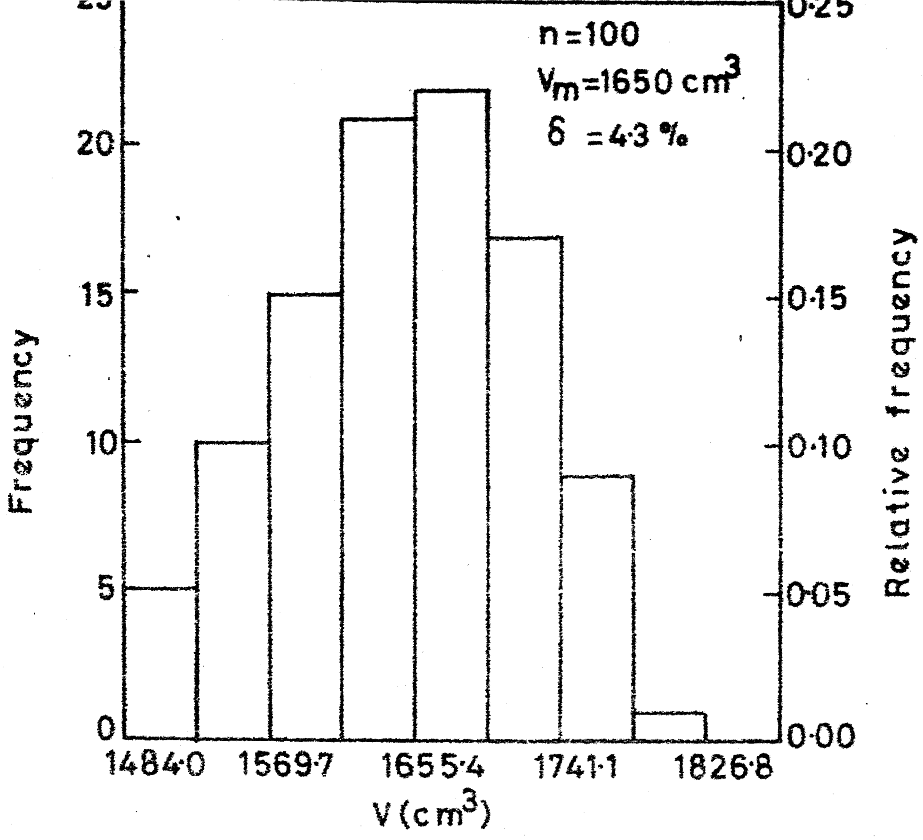


(a) Histogram

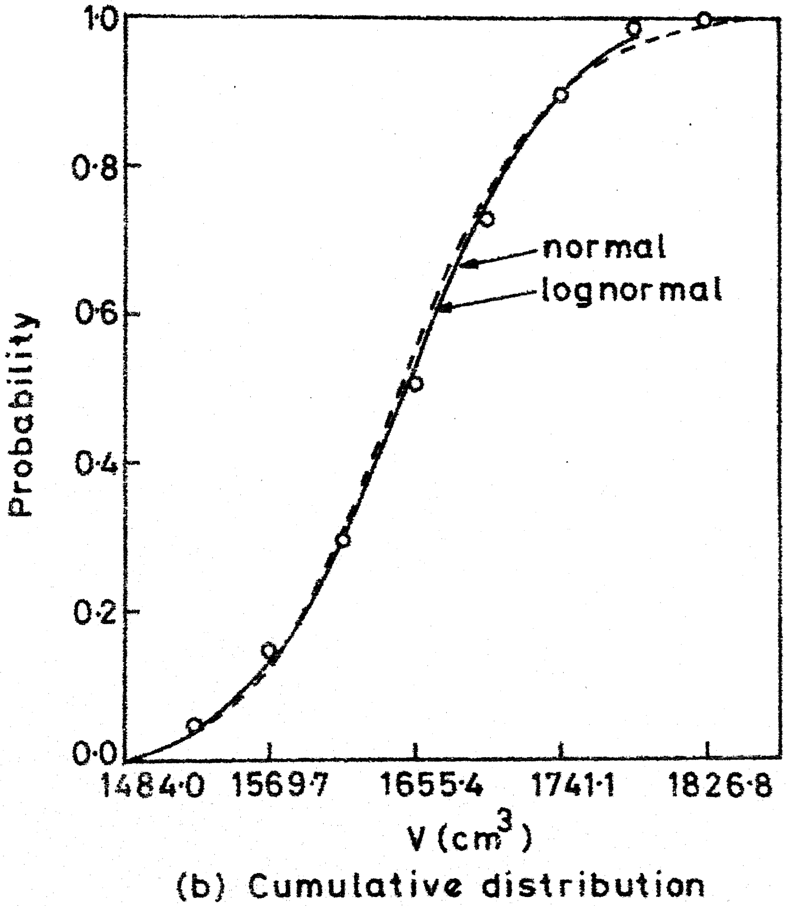


(b) Cumulative distribution

Fig. 2·8 Histogram of area of brick of fixed lot BK 8

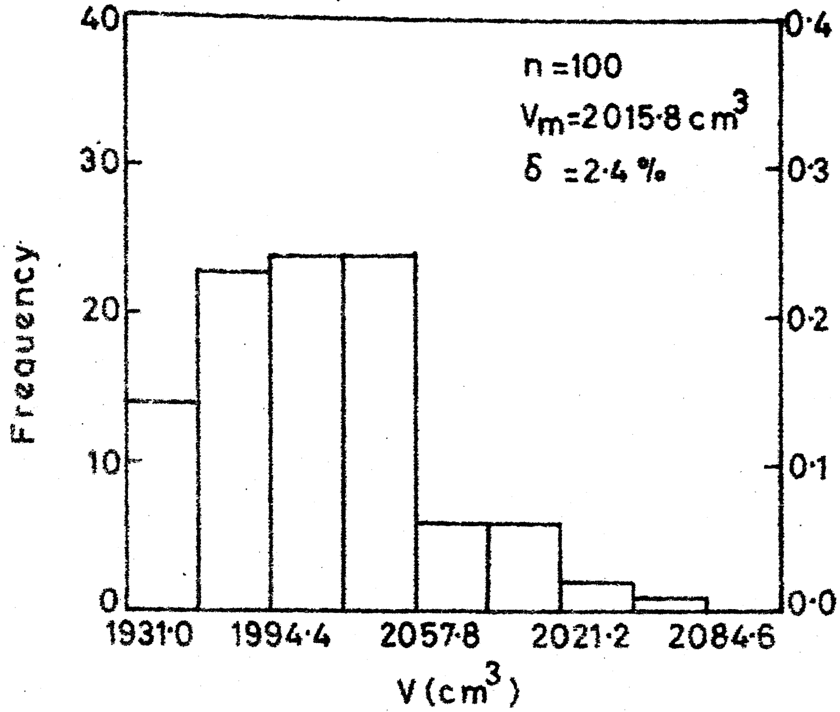


(a) Histogram

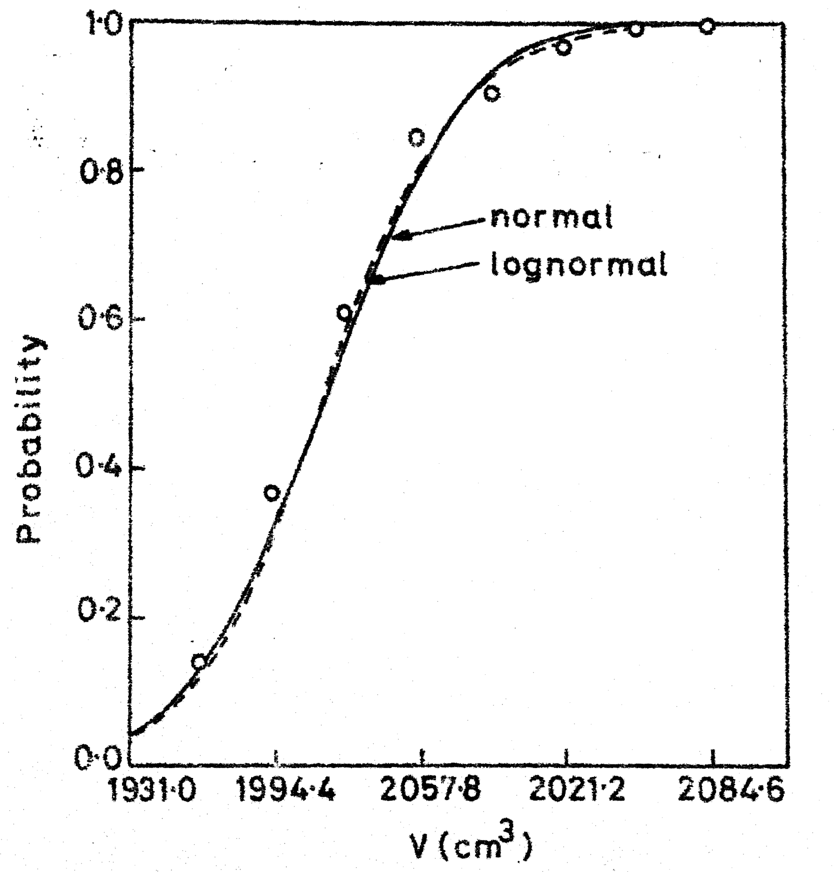


(b) Cumulative distribution

Fig. 2·9 Histogram of volume of brick of fixed lot BK1



(a) Histogram



(b) Cumulative distribution

Fig. 2·10 Histogram of volume of brick of fixed lot BK8

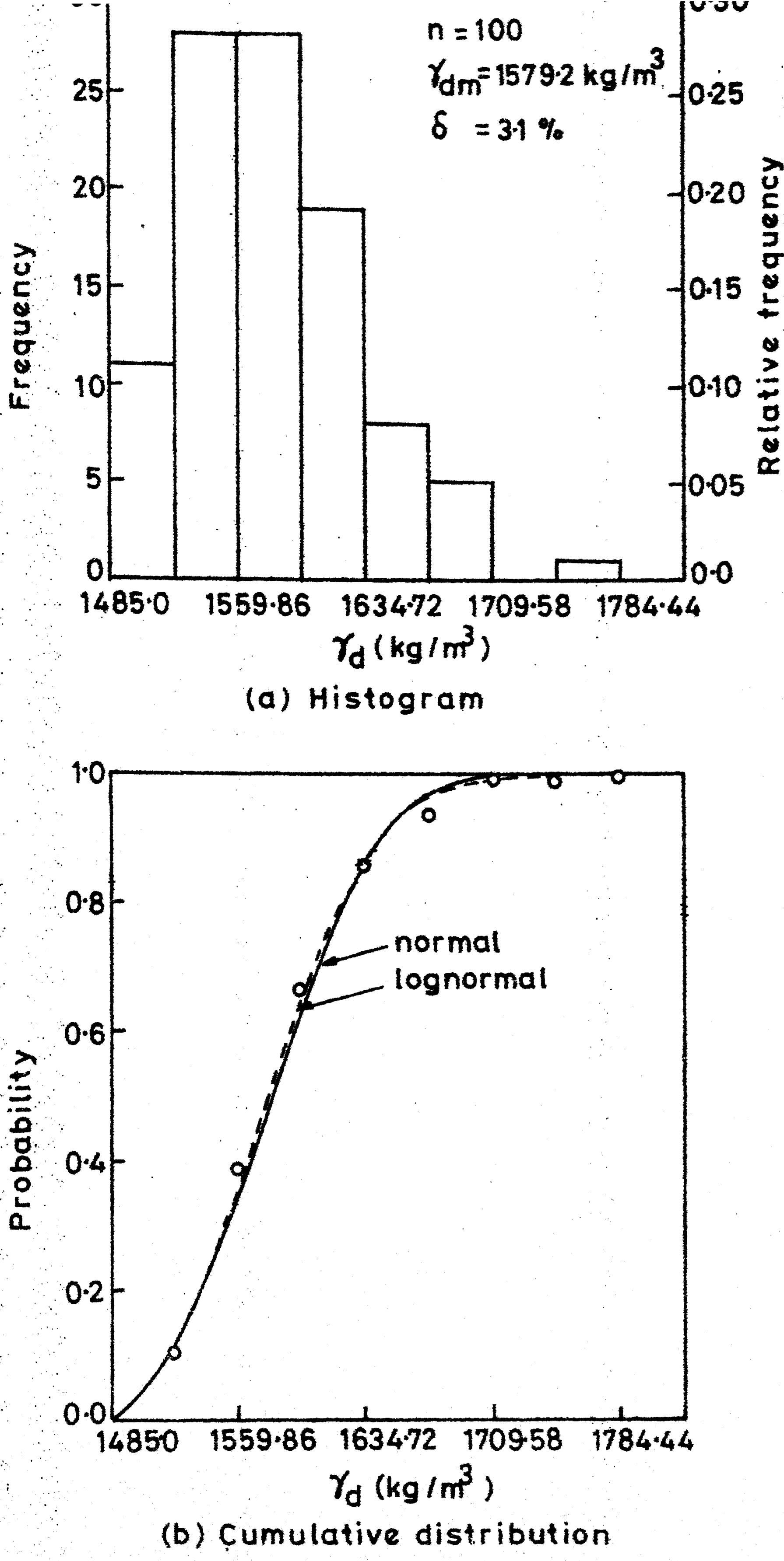
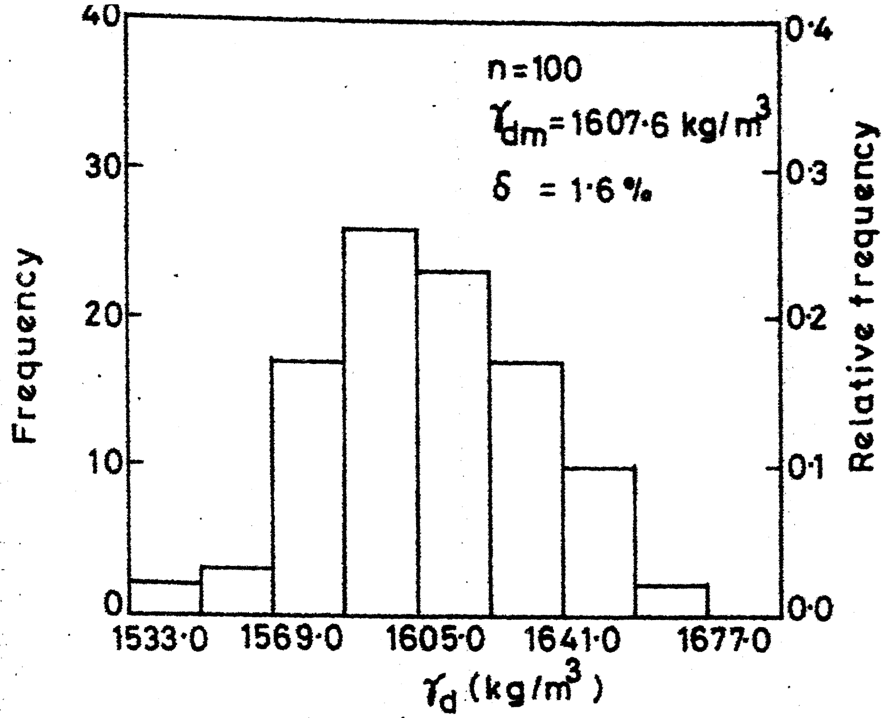
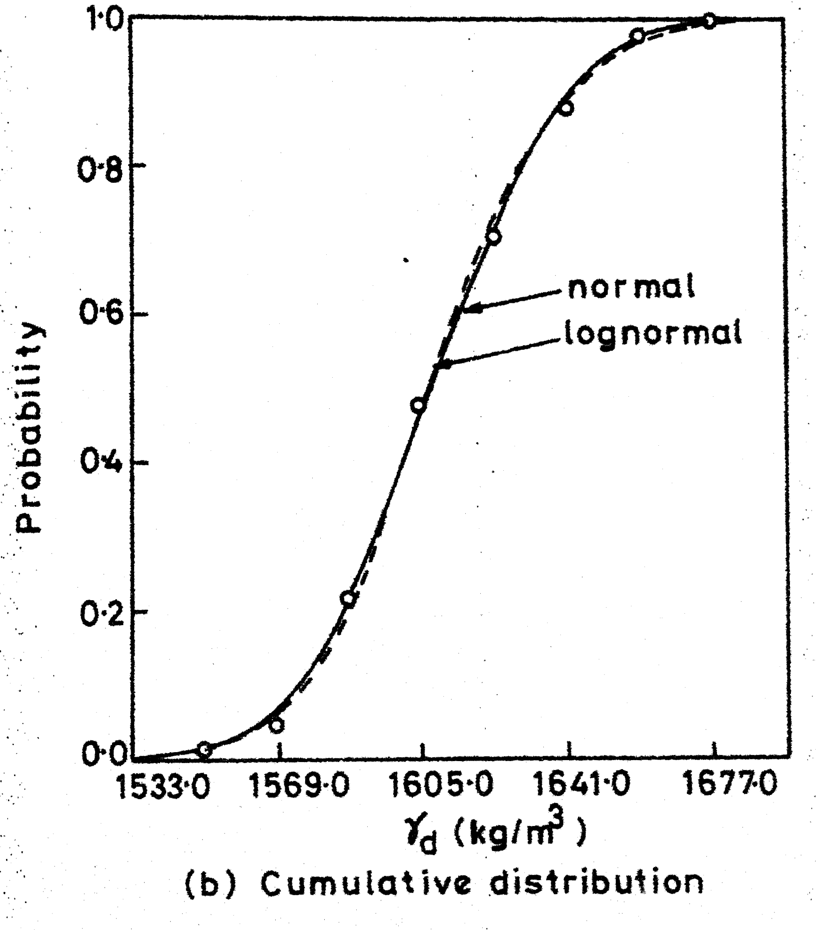


Fig.2·11 Histogram of dry density of brick of fixed lot BK1



(a) Histogram



(b) Cumulative distribution

Fig.2·12 Histogram of dry density of brick of fixed lot BK8

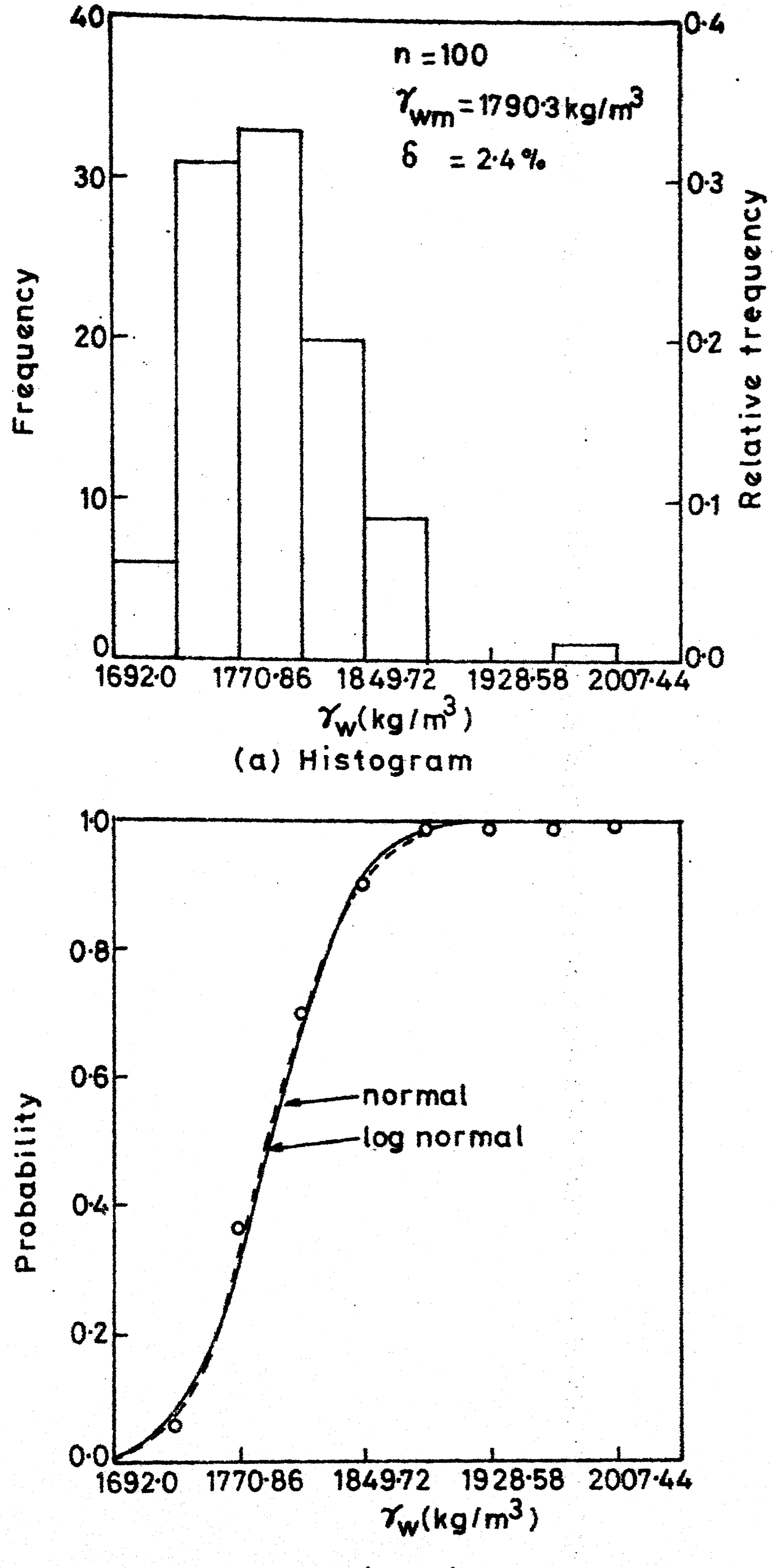
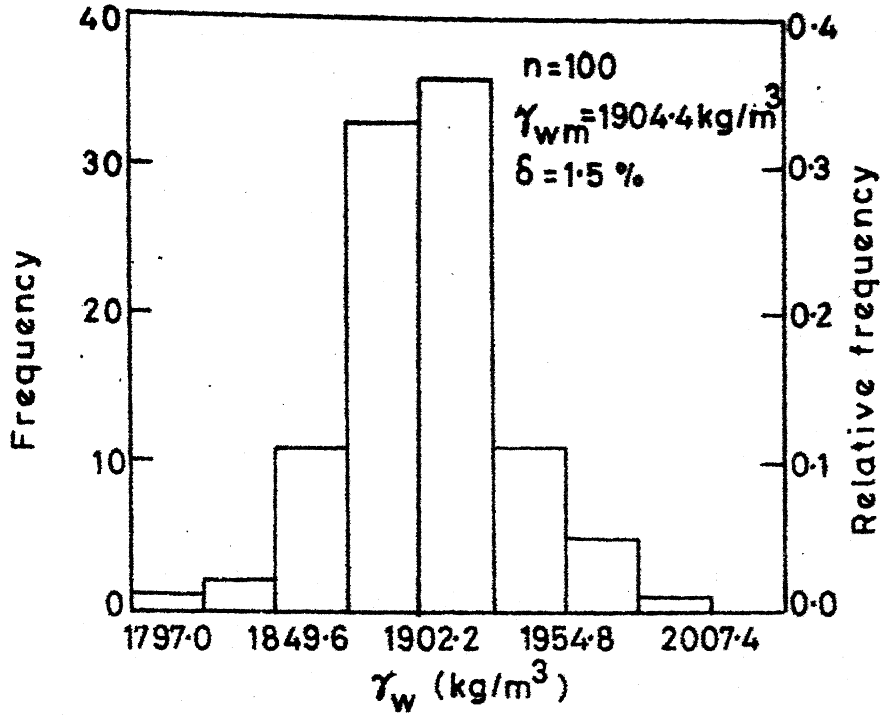
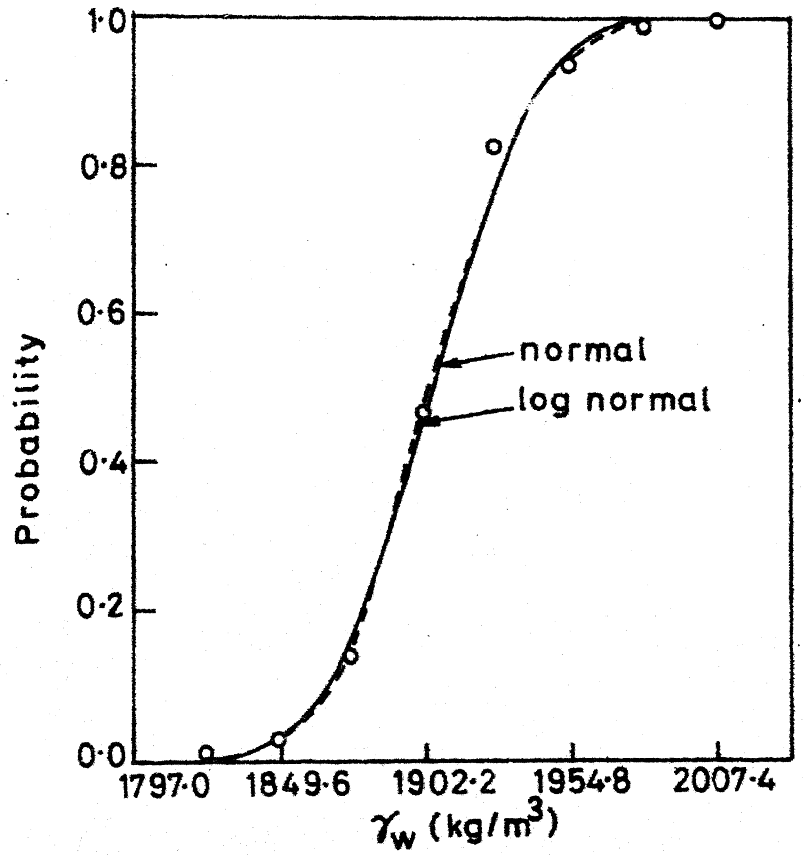


Fig. 2·13 Histogram of wet density of brick

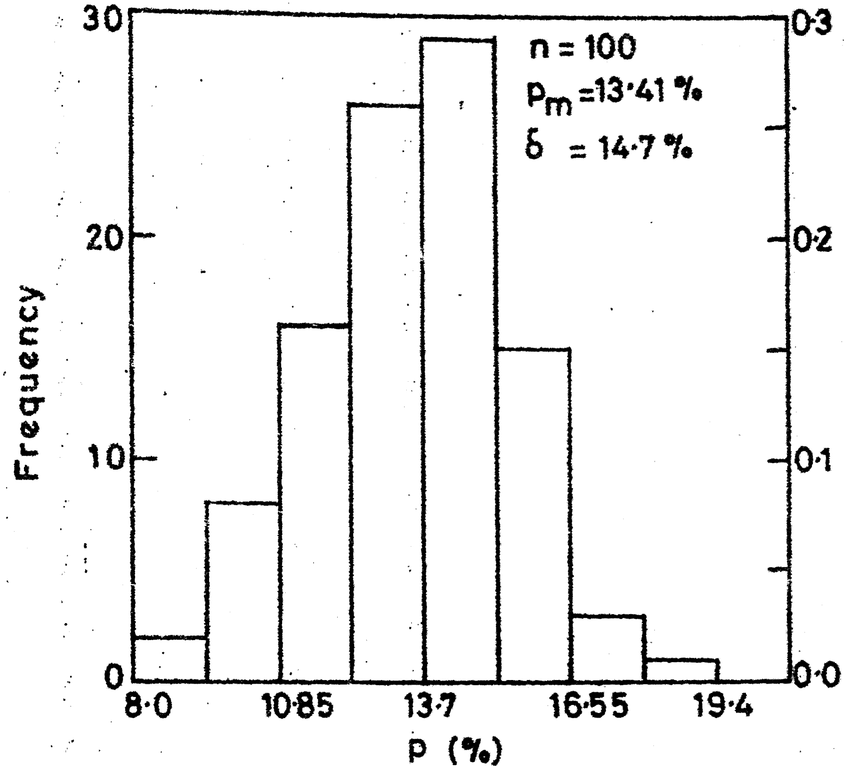


(a) Histogram

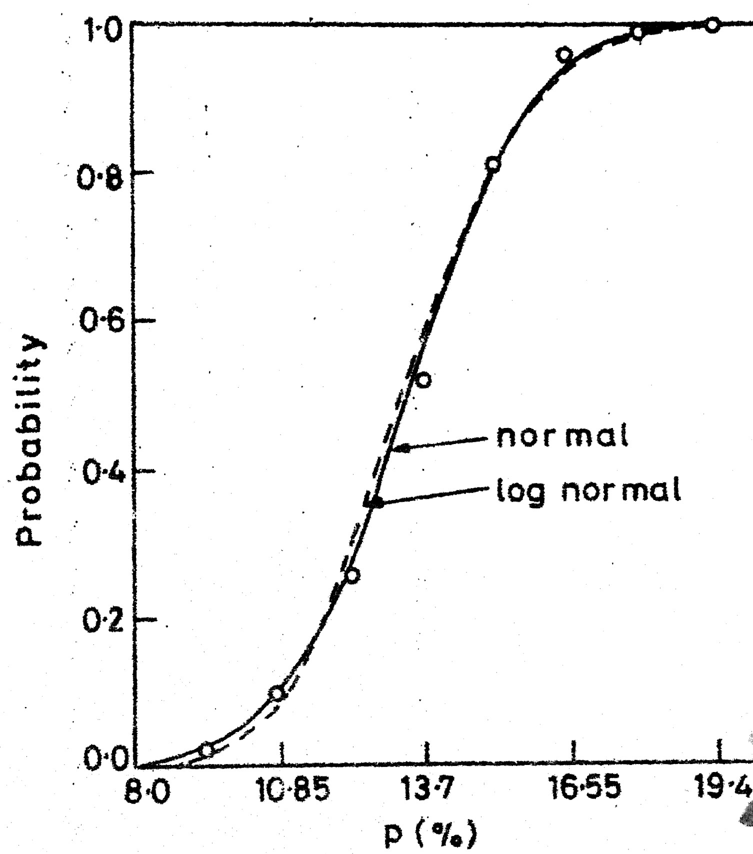


(b) Cumulative distribution

Fig. 2·14 Histogram of wet density of brick of fixed lot BK8

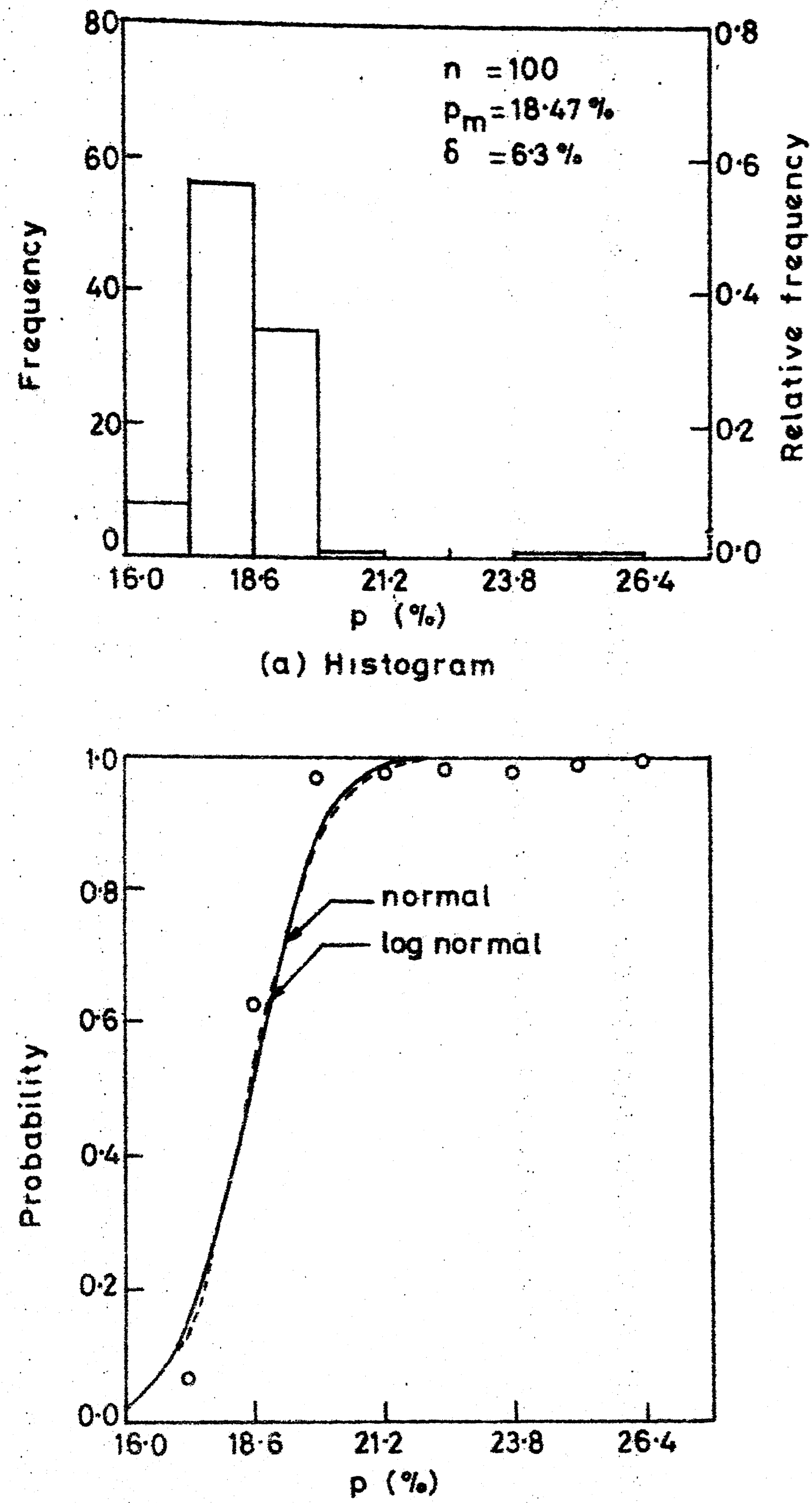


(a) Histogram



(b) Cumulative distribution

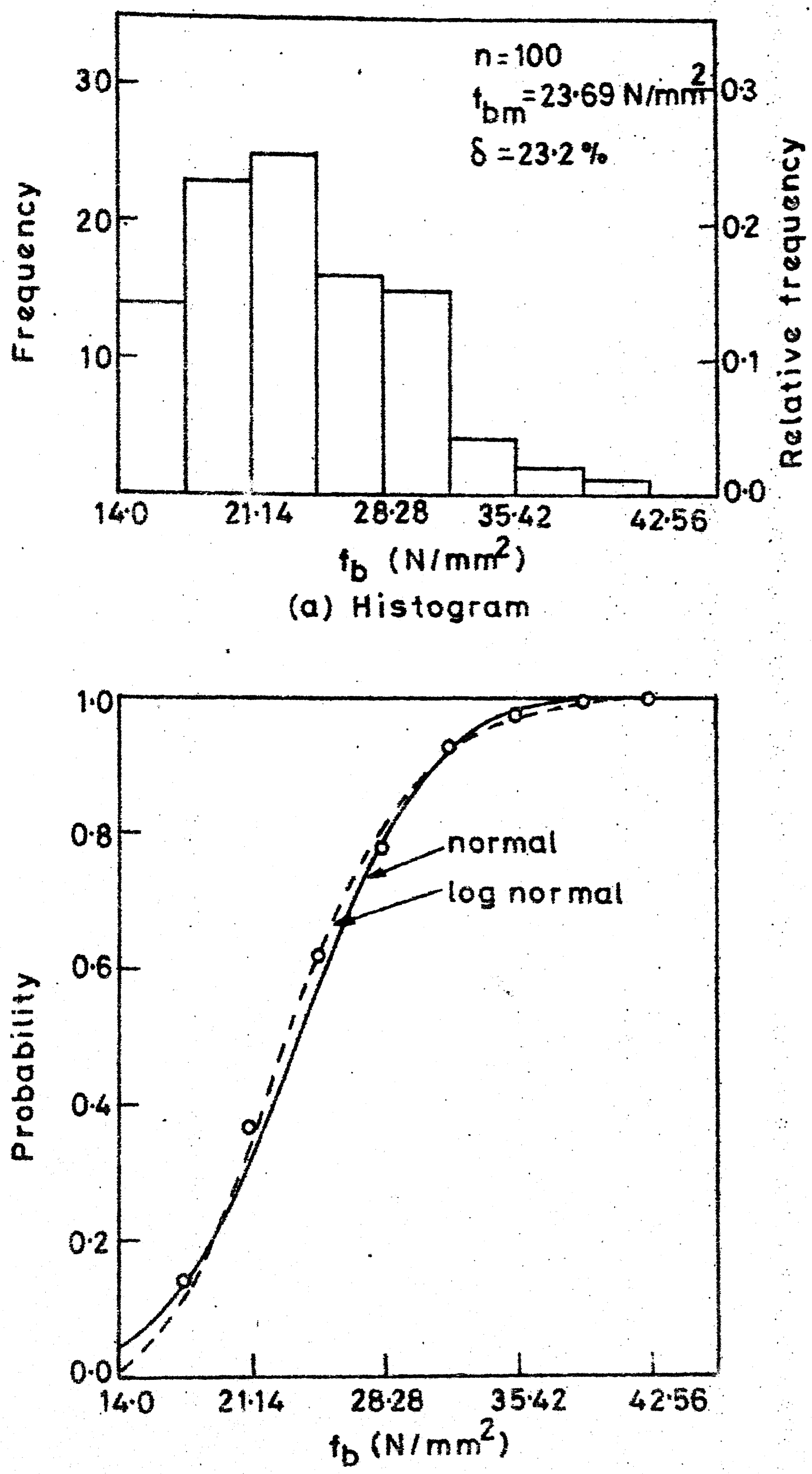
Fig. 2·15 Histogram of water absorption of brick of fixed lot BK1



(a) Histogram

(b) Cumulative distribution

Fig.2·16 Histogram of water absorption of brick of fixed lot BK 8



(a) Histogram

(b) Cumulative distribution

Fig. 2·17 Histogram of compressive strength of brick of fixed lot BK1

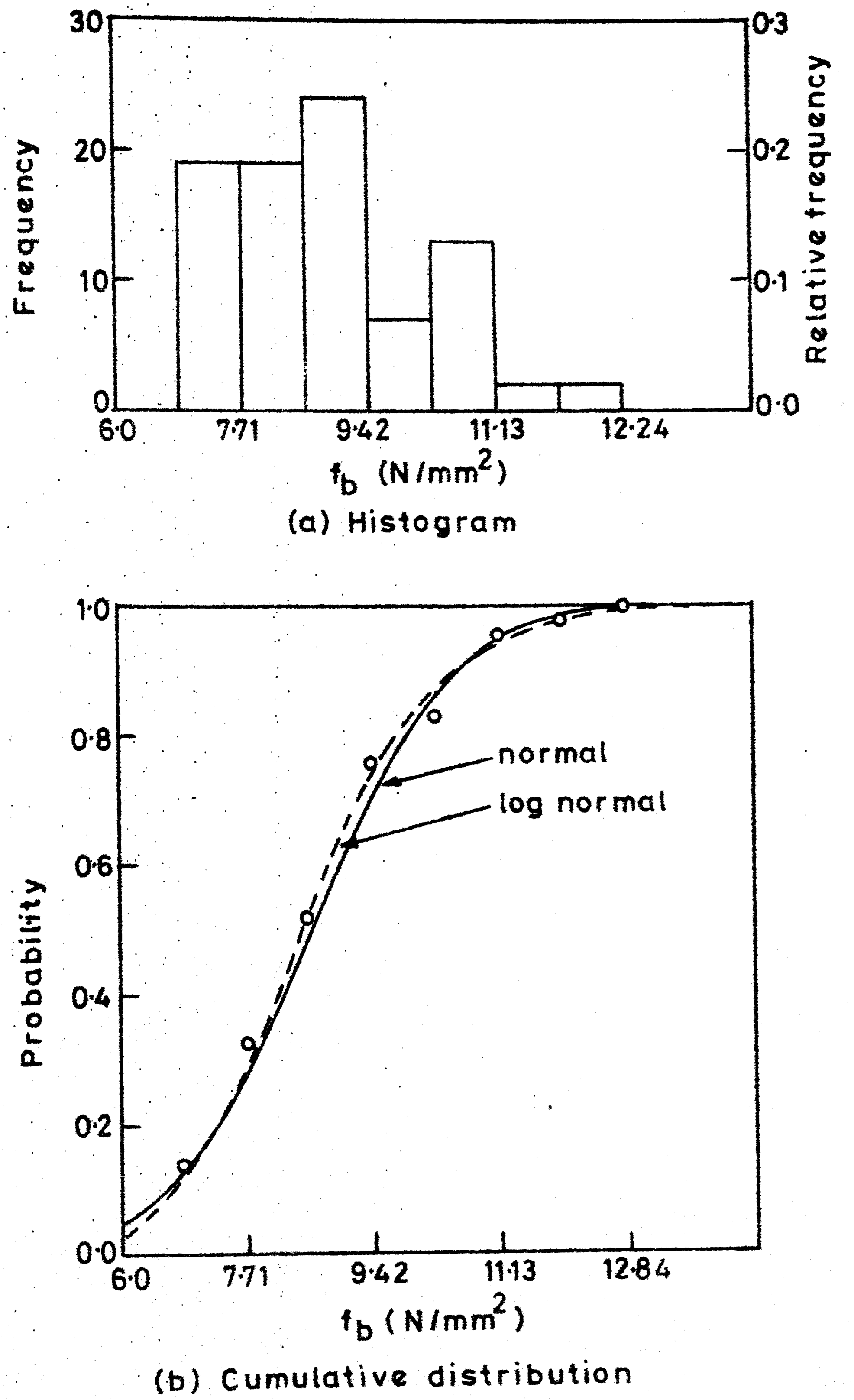


Fig. 2·18 Histogram of compressive strength of brick of fixed lot BK8

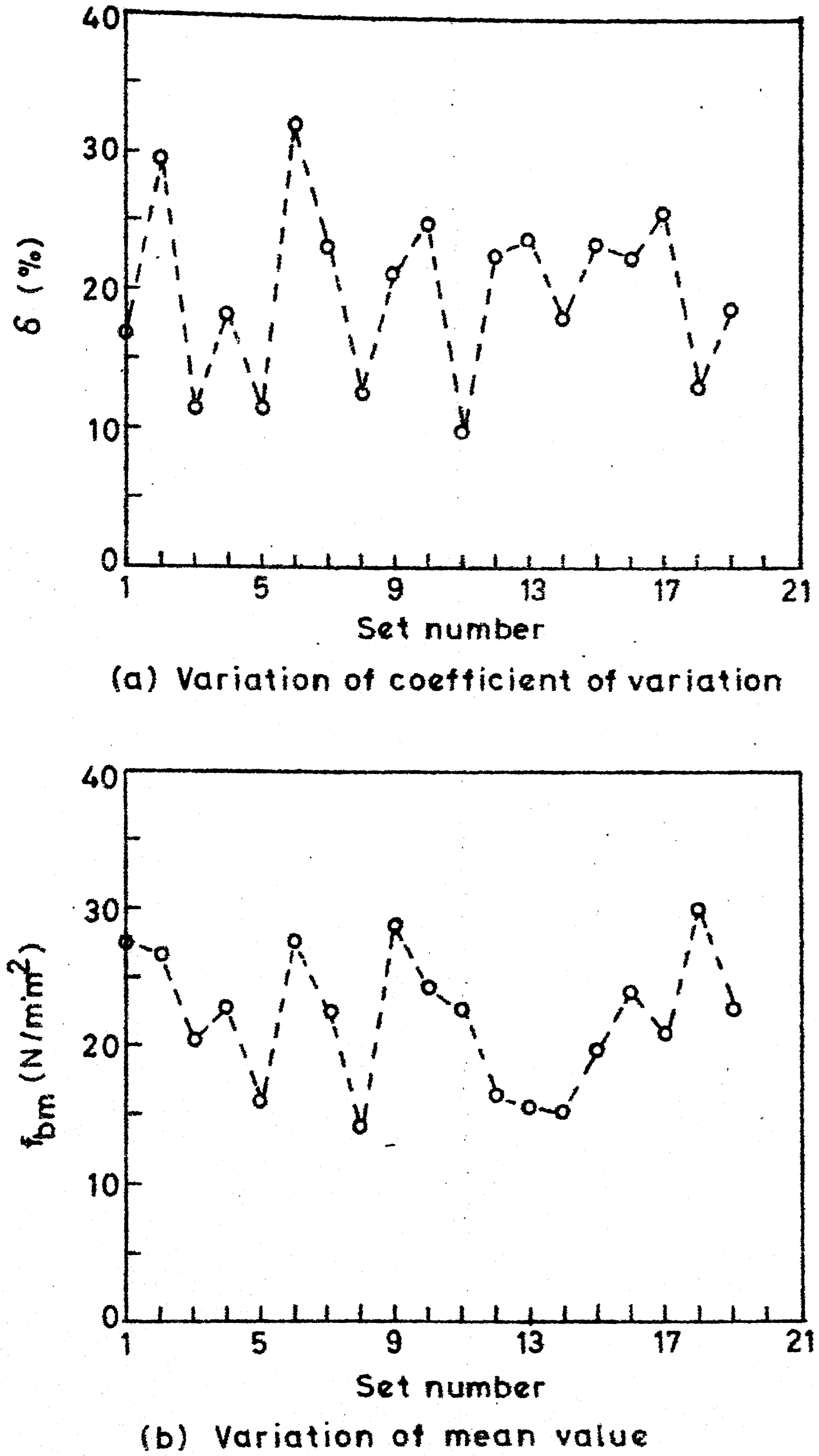
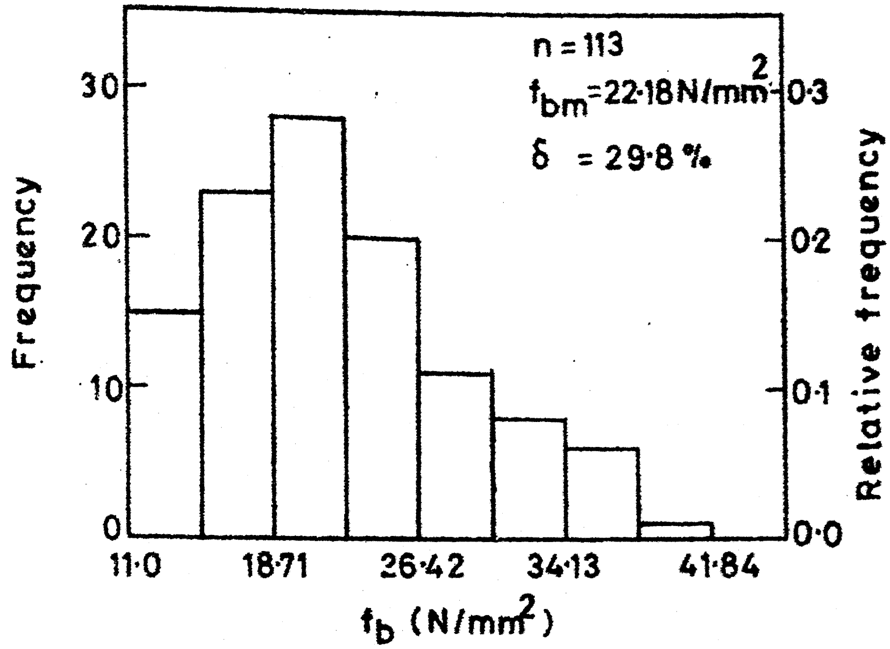
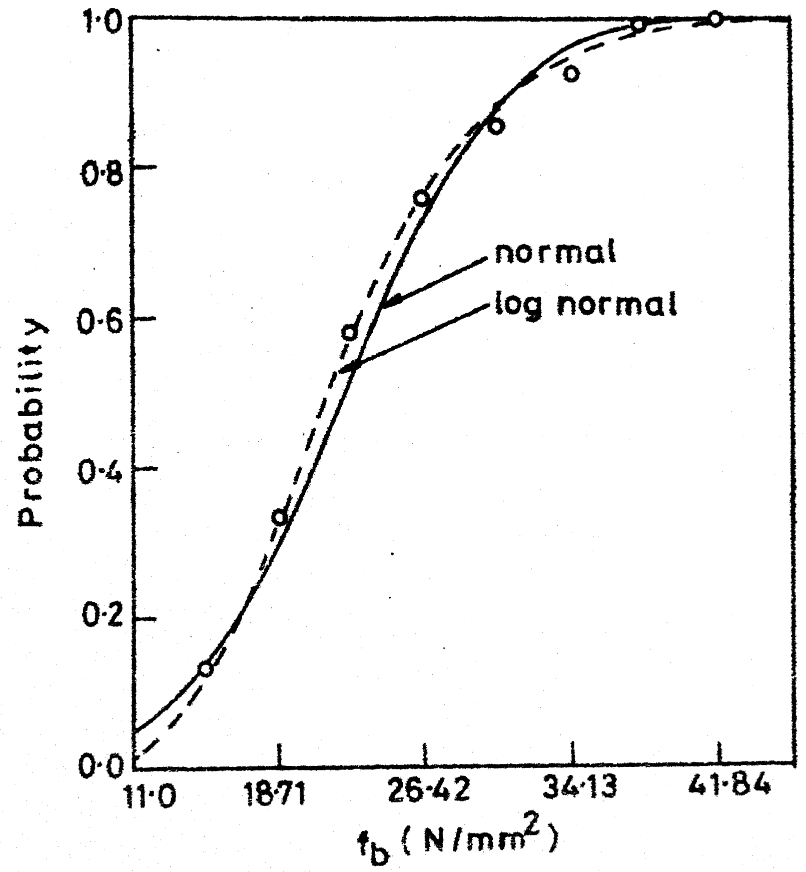


Fig. 2·19 Set variation of compressive strength of brick of random lot BK 11



(a) Histogram



(b) Cumulative distribution

Fig. 2·20 Histogram of compressive strength of brick of random lot BK11

Table 2.12 : Summary of Brick Properties

	Parameter	Mean	δ(%)	Range	
				Lower	Upper
Typical Fixed lot BK1	L	230.7	1.2	224.7	236.5
	H	64.5	2.4	60.2	68.0
	p	13.41	14.7	8.2	18.43
	f_b	23.69	23.2	13.92	39.35
Typical Fixed lot BK8	L	232.8	0.7	230.0	236.0
	H	76.1	1.6	73.5	78.5
	p	18.47	6.3	16.12	25.16
	f_b	8.58	17.6	5.54	12.40
Mixed lot 1 (BK9)	L	228.5	1.6	215.5	236.5
	H	63.3	3.3	57.3	68.8
	p	15.09	22.3	7.53	26.76
	f_b	19.89	31.0	8.05	39.69
Mixed lot 2 (BK10)	L	232.7	1.0	227.0	238.0
	H	75.7	4.8	67.5	82.4
	p	16.43	12.9	10.71	25.16
	f_b	9.84	29.8	3.84	19.53
Random lot (BK11)	L	227.2	2.1	2[illegible]1.[illegible]	241.0
	H	–	–	–	–
	p	12.05	22.8	6.00	19.4[illegible]
	f_b	22.18	29.8	11.38	38.85

available in the literature to fit a mathematical model to the observed data. Chi-square goodness of fit test is applied to find the suitability of an assumed mathematical model in the present investigation. The value of the chi-square is computed from the formula (94)

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i-E_i)^2}{E_i} \tag{2.10}$$

where

χ^2 = the value of chi-square

O_i = the observed frequency in the ith interval

E_i = the expected frequency corresponding to the assumed mathematical model in the ith interval

k = the number of categories or class intervals considered in the test.

The number of degrees of freedom is given by

$$\nu = k - r - 1 \tag{2.11}$$

where

ν = number of degrees of freedom

and r = number of parameters estimated from the data.

The chi-square value computed from Eq. 2.10 for an assumed distribution is checked with the chi-square values corresponding to different significance levels at which the assumed distribution will be accepted. The maximum significance level at which the test is satisfied, called p-level (93) is also computed. Normal and lognormal distributions are

considered in the present investigation to fit the observed data.

Normal Distribution

The probability density function of a Normal Distribution is given by (93)

$$f_X(x) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{x-X_m}{s} \right)^2 \right] \qquad -\infty \leq x \leq \infty \tag{2.12}$$

where

X_m = the mean value of random variable X

s = standard deviation.

The probability distribution function is given by

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{x-X_m}{s} \right)^2 \right] dx \tag{2.13}$$

$$= \phi\left(\frac{x-X_m}{s} \right) \tag{2.14}$$

where

$\phi(.)$ = the cumulative distribution function of a standardized normal random variable.

The parameters of the normal distribution are estimated from the data through Eq. 2.2 and Eq. 2.3 respectively.

Lognormal Distribution

The probability density function of a lognormal distribution is given by (93)

$$f_X(x) = \frac{1}{x\sqrt{2\pi}\,\sigma_{ln}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \ln m}{\sigma_{ln}}\right)^2\right]$$

$$0 \leq X < \infty \tag{2.15}$$

where m and σ_{ln} are the two parameters of the lognormal distribution. The probability distribution function is given by

$$F_X(x) = \int_0^x \frac{1}{x\sqrt{2\pi}\,\sigma_{ln}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \ln m}{\sigma_{ln}}\right)^2\right] dx \tag{2.16}$$

$$= \phi\left(\frac{\ln(x/m)}{\sigma_{ln}}\right) \tag{2.17}$$

The parameters of the lognormal distribution are expressed as

$$\sigma_{ln}^2 = \ln(\delta^2 + 1) \tag{2.18}$$

$$m = X_m \exp(-0.5\,\sigma_{ln}^2) \tag{2.19}$$

2.2.6 Statistical results

Chi-square test is carried out to fit normal distribution and lognormal distribution to the data of different parameters of the samples. The parameters of normal distribution are estimated from the data by Eqs. 2.2 and 2.3. The parameters of lognormal distribution m and σ_{ln} are estimated from the data by Eqs. 2.18 and 2.19. The maximum significance level (p-level) at which the test is satisfied is computed for the two distributions and is given in Table 2.13. Computed p-level less than 0.5 percent is not listed in the

Table 2.13 : Chi-square Test Results of Brick Properties
(p-level in percent)

Parameter		BK1	BK2	BK3	BK4	BK5	BK6	BK7	BK8	BK9	BK10	BK11
L	N	92.5	64.0	26.0	*	*	*	*	0.5	16.0	*	1.0
	LN	92.5	56.0	23.0	*	*	*	*	0.5	10.0	*	40.0
B	N	7.5	75.0	19.0	0.5	29.0	27.0	5.0	*	41.0	*	5.0
	LN	2.5	72.0	13.0	0.5	26.0	31.0	2.5	*	29.0	*	7.5
H	N	2.5	21.0	84.0	1.0	*	90.0	17.0	*	2.5	*	-
	LN	1.0	26.0	89.0	1.0	*	92.5	20.0	*	2.5	*	-
A	N	47.0	25.0	41.0	95.0	2.5	*	61.0	*	65.0	*	-
	LN	55.0	11.0	27.0	95.0	2.5	*	54.0	*	48.0	*	-
V	N	79.0	63.0	44.0	46.0	1.0	43.0	73.0	2.5	19.0	*	-
	LN	64.0	41.0	43.0	48.0	5.0	50.0	71.0	5.0	23.0	*	-
	N	12.0	25.0	17.0	53.0	*	57.0	35.0	40.0	*	*	-
	LN	21.0	61.0	25.0	41.0	*	56.0	46.0	31.0	11.0	*	-
	N	39.0	57.0	1.0	7.5	*	27.0	50.0	43.0	*	*	-
	LN	46.0	60.0	5.0	2.5	*	23.0	52.0	40.0	*	*	-
p	N	90.0	13.0	*	*	5.0	*	*	*	*	*	15.0
	LN	38.0	5.0	*	*	2.5	*	*	*	*	*	12.0
f_b	N	16.0	10.0	5.0	39.0	28.0	50.0	2.5	7.5	*	*	0.5
	LN	74.0	85.0	87.0	*	39.0	73.0	38.0	12.0	44.0	*	18.0

* p- levels is less than 0.5 percent ; N and LN stands for normal and log-normal distribution.

table. Usually a null hypothesis that the data of a particular random variable follow an assumed distribution is accepted at a level of significance 1 %, 5 % and 10 %. The p-level is computed to see whether a parameter like length, breadth etc. follows normal or lognormal distribution at a particular significance level in all the cases. Each parameter is discussed below seperately.

<u>Length</u>

Mean length of bricks of different fixed lot varied in the range of 227 to 236 mm and the coefficient of variation varied from 0.3 to 2 % . The lower and upper range is observed to be within $\pm$ 5 mm about the mean value of a particular lot. Samples of BK4, BK5, BK6, BK7 and BK8 could not be fitted to normal or lognormal distribution. The data is clubbed around the mean values which signifies that there is a deterministic trend rather than probabilistic. Lognormal seems to be a better fit over normal distribution at 1 % significance level for the data of random lot while normal distribution appears better for fixed lots BK1, BK2, BK3 and mixed lot BK9. Length of brick can be taken as normally distributed for practical purposes. The local standard of Kanpur zone is 230 mm. The probability of getting length of brick less than 230 mm can be obtained as (from mixed lot 1),

$$P\ (L \leq 230) = \phi\left(\frac{230-228.5}{3.656}\right) = \phi\ (0.41)$$
$$= 0.658$$

which is same as 658 in 1000 bricks (assuming normal distrubution) . The probability of getting length of brick less than 230 mm (from mixed lot 2),assuming normal, is

$$P\ (L \leq 230) = \phi\left(\frac{230-232.7}{2.327}\right) = \phi(-1.16) = 0.123$$

which is same as 123 in 1000 bricks.

Breadth

Mean breadth of bricks of different fixed lots varied from 108 mm to 119 mm and the coefficient of variation varied from 0.5 to 2.8 percent. Normal distribution fitted well at 5 percent level of significance except in two cases. Normal also fitted the random lot at 5 percent level. Although lognormal appears better in case of random lot, normal distribution seems to be a better fit in most of the cases. Thus breadth of brick can be taken as normally distributed random variable. Assuming the local standard of breadth of bricks 110mm (for mixed lot 1)

$$P\ (B \leq 110\ ) = \phi\left(\frac{110-109.4}{2.5162}\right) = \phi(0.24) = 0.595$$

which is same as 595 in 1000 bricks.

For mixed lot 2, the probability of getting breadth of bricks less than 110 mm is

$$P\ (B \leq 110) = \phi \left(\frac{110-116}{2.668} \right) = \phi\ (-2.25) = 0.012$$

which is same as 12 in 1000 bricks.

Height

Mean height of bricks of different manufacturers of Kanpur zone (BK1, BK2 and BK3 lot) varied from 62 mm to 64.5 mm and the coefficient of variation varied from 2.4 to 3.4 percent. The coefficient of variation of height of bricks appears to be higher as compared to length and breadth of bricks. Mean height of bricks of different manufacturers of Faroke zone varied from 70.5 mm to 79.7 mm and the coefficient of variation varied from 1.2 to 2.1 percent. Normal fits the data at 1 percent level except in few cases.

The local standard of height of bricks is 63 mm in Kanpur zone, and the probability of getting less than 63 mm (from mixed lot 1) is

$$P(H \leq 63) = \phi \left(\frac{63-63.3}{63.3 \times 0.033} \right) = \phi\ (-0.144)$$
$$= 0.444$$

which is same as 444 in 1000 bricks.

The local standard of height of bricks in Faroke zone is 75 mm, and the probability of getting less than 75 mm (from mixed lot 2) is

$$P(H \leq 75) = \phi \left(\frac{75-75.7}{75.7 \times 0.048} \right) = \phi\ (-0.193)$$
$$= 0.424$$

which is same as 424 in 1000 bricks.

Area

The cross-sectional area of bricks is observed to have higher variability as compared to the individual variability of length and breadth. The coefficients of variation of area of bricks are found to be 3.3 percent and 2.4 percent for mixed lot 1 and 2 respectively. In both the cases, a marginal negative skewness is observed. Normal and lognormal distribution fits the data of all the fixed lots (except BK6 and BK8) at 1 percent significance level. The local standard of bricks of Kanpur zone, is

$$L = 230 \text{ mm}, \quad B = 110 \text{ mm} \quad \text{and} \quad H = 63 \text{ mm}$$

$$A = 230 \times 110 = 25300 \text{ mm}^2 = 253 \text{ cm}^2$$

Probability of getting area of bricks less than 253 cm^2 is

$$P(A \leq 253) = \phi\left(\frac{253-250.1}{250.1 \times 0.003}\right) = \phi(-0.35)$$
$$= 0.363$$

which is same as 363 in 1000 bricks (assuming normal).

Similarly for Faroke (Kerala) zone, the local standard is

$$L = 230 \text{ mm}, \quad B = 110 \text{ mm and} \quad H = 75 \text{ mm}$$

$$A = 25300 \text{ mm}^2 = 253 \text{ cm}^2$$

Thus, $$P(A \leq 253) = \phi\left(\frac{253-270.1}{270.1 \times 0.024}\right) = \phi(-2.64)$$
$$= 0.004$$

which is same as 4 in 1000 bricks (assuming normal).

Volume

The coefficient of variation of volume of bricks are found to be 5.5 percent and 6.2 percent for mixed lot 1 and 2 respectively. Normal distribution fits the data of all the fixed lot at 1 percent significance level whereas lognormal fits at 5 percent. Lognormal distribution is found to be a better fit at 1 percent significance level in some of the cases.

The volume of a brick as per local standard of Kanpur zone is

$$V = 230 \times 110 \times 63 = 1593900 \text{ mm}^3 = 1593.9 \text{ cm}^3$$

Probability of getting volume of bricks less than 1593.9 cm^3, assuming normal distribution, is given by (for mixed lot 1)

$$P\ (V \leq 1593.9) = \phi\ \left(\frac{1593.9 - 1584.3}{1584.3 \times 0.055}\right) = \phi\ (0.11)$$

$$= 0.543$$

which is same as 543 in 1000 bricks.

Similarly, volume of a brick as per local standard of Faroke (Kerala) zone is

$$V = 230 \times 110 \times 75 = 1897500 \text{ mm}^3 = 1897.5 \text{ cm}^3$$

and $$P\ (V \leq 1897.5) = \phi\ \left(\frac{1897.5 - 2045.8}{2045.8 \times 0.062}\right) = \phi\ (-1.17)$$

$$= 0.121$$

which is same as 121 in 1000 bricks.

Dry Density

The coefficients of variation of dry density of bricks are found to be 6.2 percent and 5.5 percent for mixed lot 1 and 2 respectively. Normal distribution fits the data of all the fixed lots (except BK5) at 10 percent significance level. Lognormal distribution also fits at 10 percent significance level except BK5 lot. In some of the lots lognormal appears a better fit over normal distribution and vice versa.

Wet Density

The coefficients of variation of wet density of bricks are found to be 4.8 percent and 3.9 percent for mixed lot 1 and 2 respectively which is slightly lower as that of dry density. Mixed lots did not fit to any of the distribution whereas fixed lots, except BK5, fit normal and lognormal distributions at 1 percent significance level.

Percent of Water Absorption

The mean value of different fixed lots varied between 13.41 to 18.47 with coefficient of variation varying from 4.7 percent to 17.7 percent. Percent of water absorption of mixed lot 1 has a mean value 15.09 with a coefficient of variation 22.3 percent whereas mixed lot 2 has mean 16.43 and coefficient of variation 12.9 percent. Mean of percent

water absorption is found to be 12.05 (ranging from 6.0 to 19.4) with a C.O.V. = 22.8 percent for the random lot. Both normal and lognormal did not fit even at 0.5 percent significance level except in three cases.

Brick Strength

Mean compressive strength of brick of fixed lots BK1, BK2 and BK3 is in the range of 14.57 N/mm^2 to 23.69 N/mm^2 with C.O.V. varying from 22.5 percent to 24.6 percent. Mean brick strength of fixed lots BK4, BK5, BK7 and BK8 is in the range of 7.6 N/mm^2 to 14.16 N/mm^2 with C.O.V. varying from 15.1 percent to 18.6 percent. Mean value of mixed lot 1 and mixed lot 2 are 19.89 N/mm^2 and 9.84 N/mm^2 which clearly represents that the quality of bricks of Kanpur zone are better than that of Faroke (Kerala) zone. The coefficient of variation of mixed lot 1, mixed lot 2 and random lot are found to be around 30 percent which signifies variability in brick manufacturing from manufacturer to manufacturer. Normal distribution fits the data of all the fixed lots at 2.5 percent significance level while log-normal could not be fitted to the data of fixed lot BK4 even at 0.5 percent significance level. Normal did not fit mixed lots and random lot at 1 percent significance level. Thus, normal distribution can be accepted at 2.5 percent significance level to represent the probability distribution of compressive strength of brick.

2.3 Statistical Analysis of Thickness of Mortar Joint

The thickness of mortar joint has two aspects in masonry construction, (i) aesthetic aspect and (ii) strength aspect. Too much variability of joint thickness in a wall does not give a good appearence to the observer or owner of the building. The thickness of mortar joints play a great role on the strength of brickwork. Tests carried out by Francis et al. (25) showed that prism compressive strength is a function of average joint thickness and reduces as the joint thickness increases. The strength of prism decreases at a faster rate with increase in joint thickness for perforated bricks as compared with the solid bricks. So, it is expected that the variability of joint thickness will affect the strength of brickwork although no such theoretical relationship is established. To study the variability of joint thickness, data is collected from different existing structures and statistical analysis is carried out, and is presented in the following sections.

2.3.1 Collection of field data and notations

Three different buildings of exposed brick joint wall were selected for study. Horizontal joint (called bed joint) thickness and vertical joint thickness were measured at different locations of the wall selected at random. 100 measurements were carried out for each bed joint and vertical joint thickness of each building. Thickness was measured by a

slide calipers from exposed surface of the wall upto an accuracy of 0.02 mm. The three sets of data for bed joint thickness and vertical joint thickness were combined for the statistical analysis of the combined sets. The following notations are used in the subsequent discussions.

Building No.	Notation	Remarks
Building 1	MTH1	Bed joint
	MTV1	Vertical joint
Building 2	MTH2	Bed joint
	MTV2	Vertical joint
Building 3	MTH3	Bed joint
	MTV3	Vertical joint
Bed joint data of three buildings combined	MTHC	Bed joint
Vertical joint data of three buildings combined	MTVC	Vertical joint

One hundred measurements of bed joint thickness of mortar joint were taken after breaking some brick columns. These are the actual joint thicknesses inside the columns and are refered in the text as MTH4.

2.3.2 Analysis of data

The data of bed joint thickness and vertical joint thickness are analysed and the result is given in Table 2.14.

Table 2.14: Statistical Analysis of Thickness of Mortar Joint

Designation	n	Mean	δ(%)	C_s	C_k	Range Lower	Range Upper
MTH1	100	13.48	12.4	0.552	2.879	10.50	17.60
MTH2	100	13.02	12.8	0.261	2.586	9.82	17.00
MTH3	100	12.28	14.0	0.124	2.986	8.80	16.50
MTV1	100	12.21	14.2	0.770	3.255	9.50	17.00
MTV2	100	11.78	19.5	0.421	2.499	7.50	17.50
MTV3	100	12.10	15.1	0.073	2.434	8.50	16.00
MTHC	300	12.93	13.5	0.245	3.005	8.80	17.60
MTVC	300	12.03	16.4	0.340	2.767	7.50	17.50
MTH 4	100	14.90	11.5	-0.086	2.736	10.60	19.00

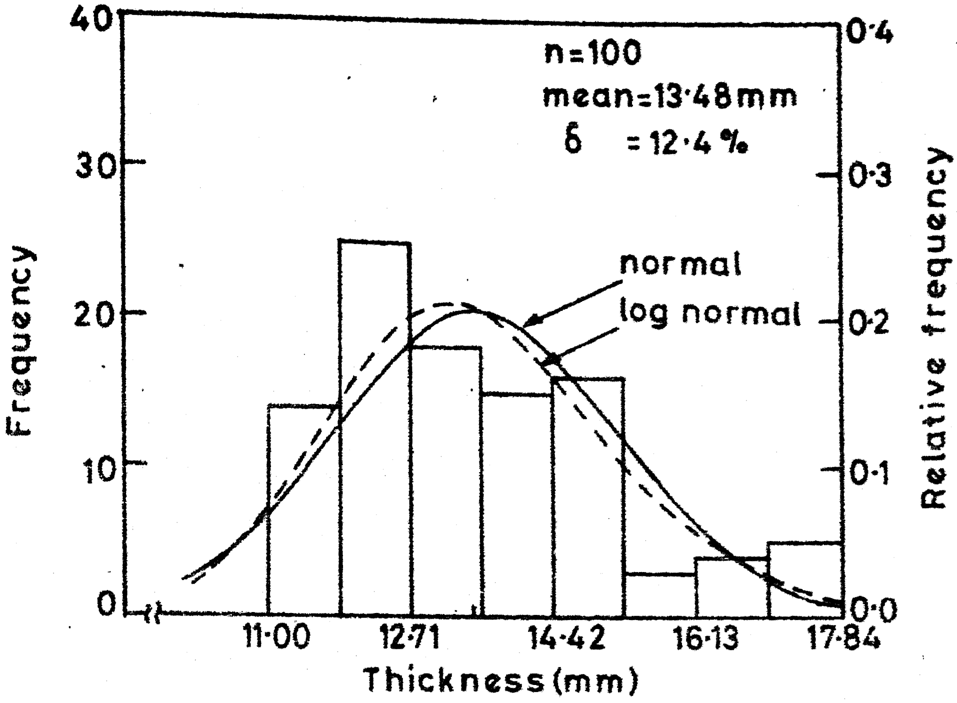
The coefficient of variation of bed joint thickness varied between 12 to 14 percent in all the buildings whereas that of vertical joint thickness varied from 13 to 20 percent. The lower and upper range has a deviation of $\pm$ 5 mm from the mean in all the cases. Chi-square test for normal and lognormal distribution is conducted and the result of chi-square test is given in Table 2.15. All the data satisfies lognormal distribution at a level of significance of 2.5 percent whereas all of them satisfies normal distribution at a level of significance of 1 percent. Histogram and the fitted distribution of two typical bed joint thickness (MTH1) and (MTH4) are given in Fig. 2.21(a) and Fig. 2.21(b) respectively.

2.4 Statistical Analysis of Strength of Mortar

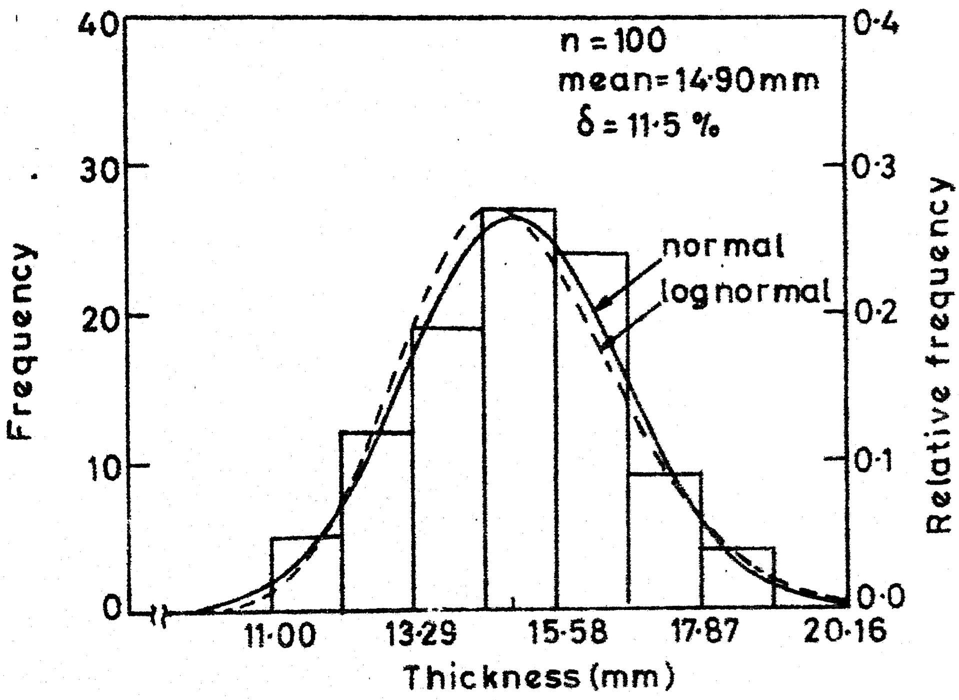
Mortar is used as the binding material between brick units. The mix proportion of cement sand mortar in load bearing brick work and reinforced brickwork varies from 1:3 (cement:sand) to 1:6. The water cement ratio varies in the range of 0.7 to 1.0 and it is usually governed by the requirement of mortar. Although there is considerable emphasis on the quality maintenance of bricks, there is hardly any qualitative test carried out in the field on the strength of mortar. The cement sand mortar needed for two to four hours of working period is prepared at one time and the

Table 2.15: Chi-square Test Result of Thickness of Mortar Joint

Designation	Normal					Log Normal				
	X_m	s	χ^2	ν	p-level (%)	m	σ_{ln}	χ^2	ν	p-level (%)
MTH1	13.48	1.67	16.80	6	1.0	13.38	0.123	10.55	6	10.0
MTH2	13.02	1.67	4.35	6	62.0	12.91	0.128	4.18	6	65.0
MTH3	12.28	1.71	10.84	6	7.5	12.16	0.139	14.01	6	2.5
MTV1	12.21	1.73	16.54	6	1.0	12.09	0.141	9.72	6	13.0
MTV2	11.78	2.29	12.83	6	2.5	11.56	0.193	8.53	6	20.0
MTV3	12.10	1.83	7.87	6	24.0	11.96	0.150	13.40	6	2.5
MTHC	12.93	1.75	15.86	8	2.5	12.81	0.135	15.52	8	2.5
MTVC	12.03	1.97	8.62	8	37.0	11.87	0.163	5.88	8	66.0
MTH4	14.90	1.71	2.21	6	89.0	14.80	0.115	4.90	6	55.0



(a) Bed joint (MTH 1)



(b) Bed joint (MTH 4)

Fig. 2·21 Histogram of thickness of mortar joint

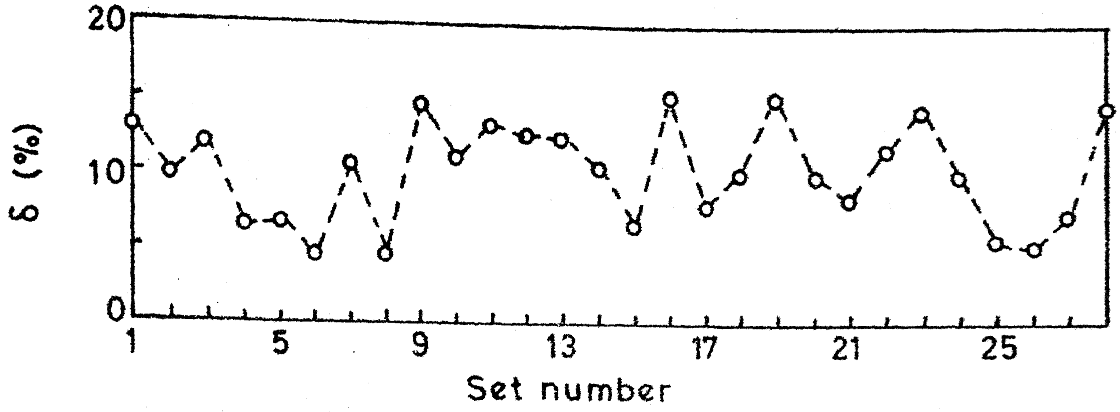
consistency of mortar is maintained by constant addition of water and remixing the material. This is a common practice observed in India even in large brickwork construction. The strength of masonry depends on the quality of bricks and mortar used in the construction. A study of the variability of mortar strength is needed in order to understand the behaviour of brickwork strength. Due to the absence of data from field, mortar cubes were cast, cured and tested in the laboratory after 28 days. The statistical analysis of mortar strength data is presented in subsequent sections.

2.4.1 Sample collection

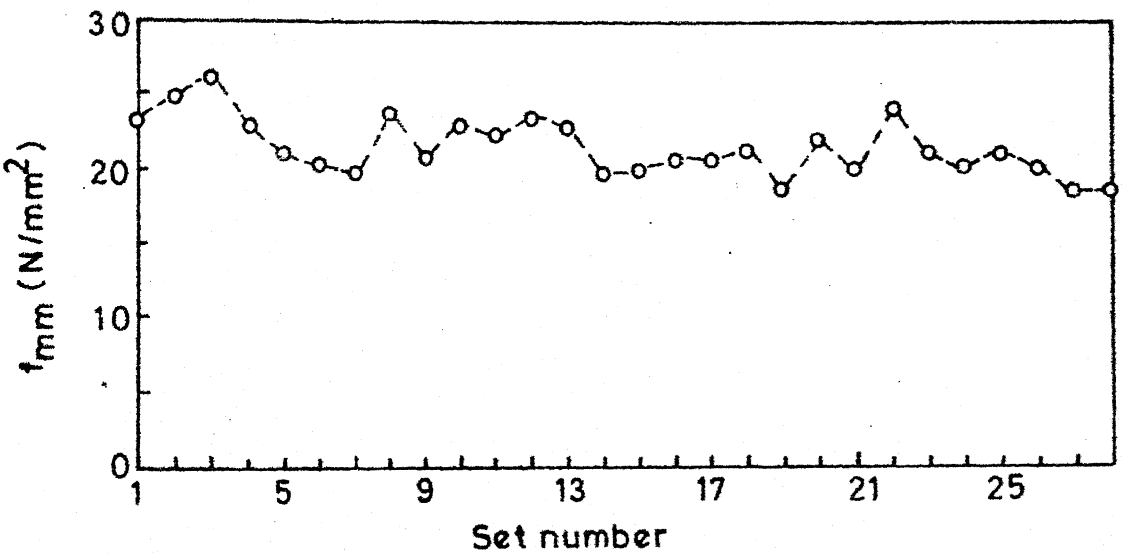
Mortar cubes of 70.7 mm size were cast for three different mixes i.e., 1:3 , 1:4 and 1:5 (cement:sand by weight) and tested after 28 days of water curing in the laboratory. Six cubes were cast at a time for each mix and is called a set. A fresh mix has been done for casting every six cubes. The amount of water is kept constant at each set of a particular mix.

2.4.2 Histogram and statistical analysis

The variation of mean value and coefficient of variation of compressive strength of mortar f_m (N/mm^2) of different sets of each mix is shown in Fig. 2.22 through Fig. 2.24. All the sets of a particular mix is combined and statistical analysis is done. The statistical analysis of

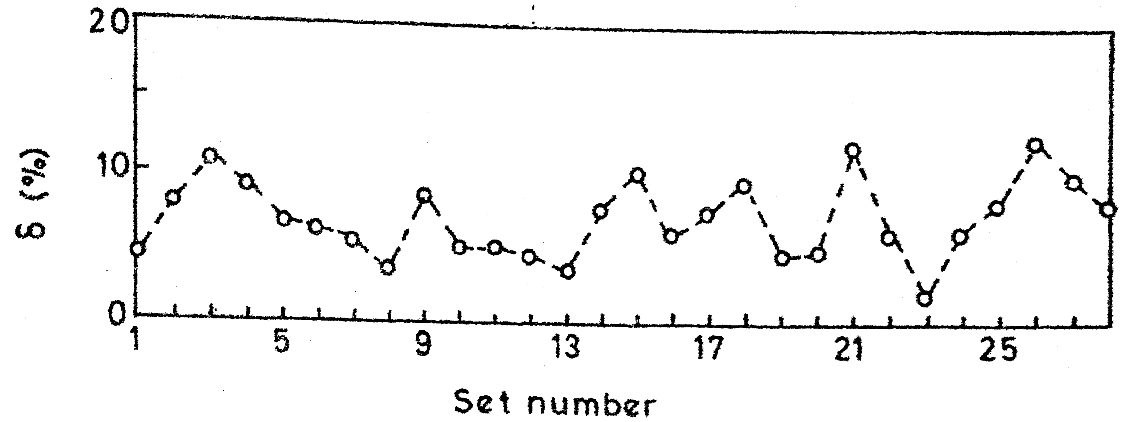


(a) Variation of coefficient of variation

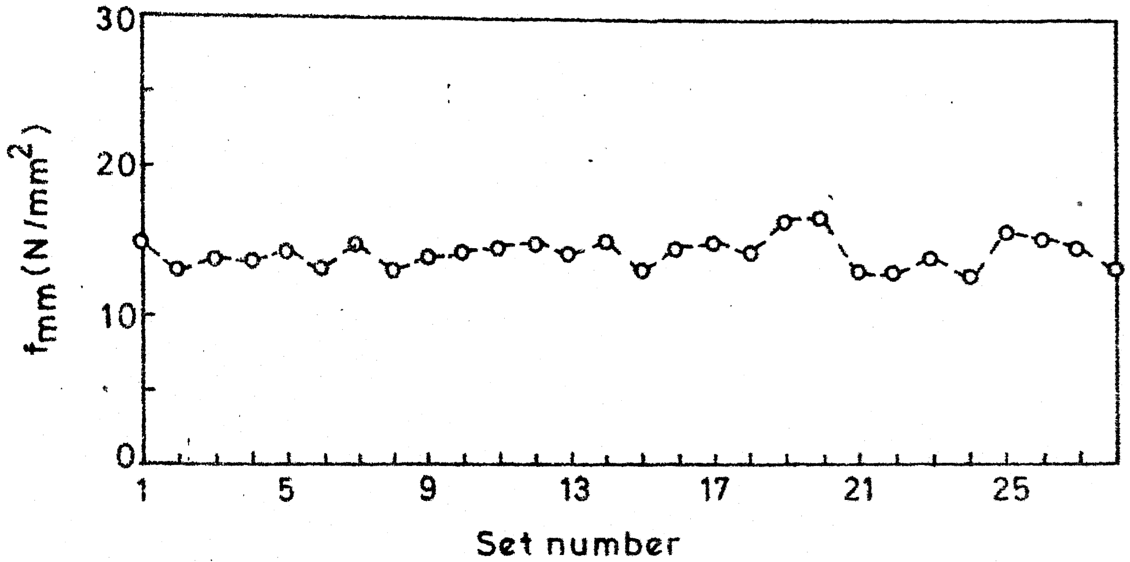


(b) Variation of mean value

Fig. 2·22 Set variation of strength of mortar (1:3 mix)

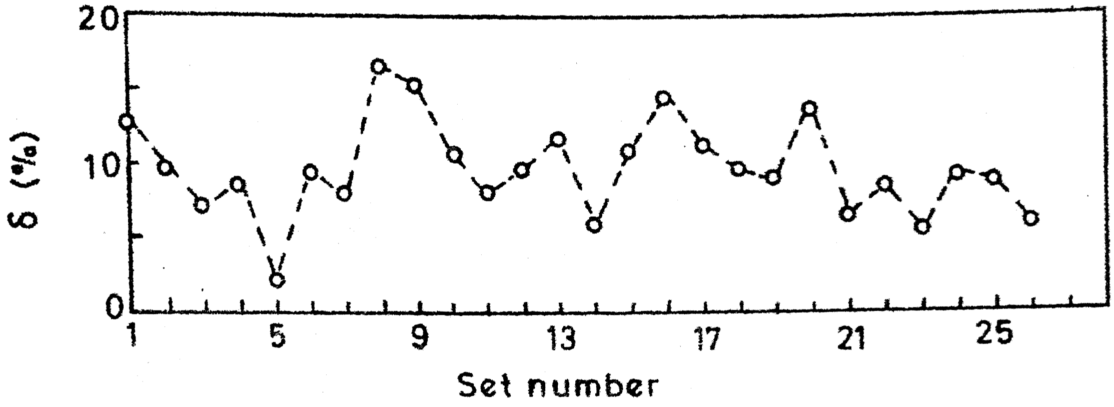


(a) Variation of coefficient of variation

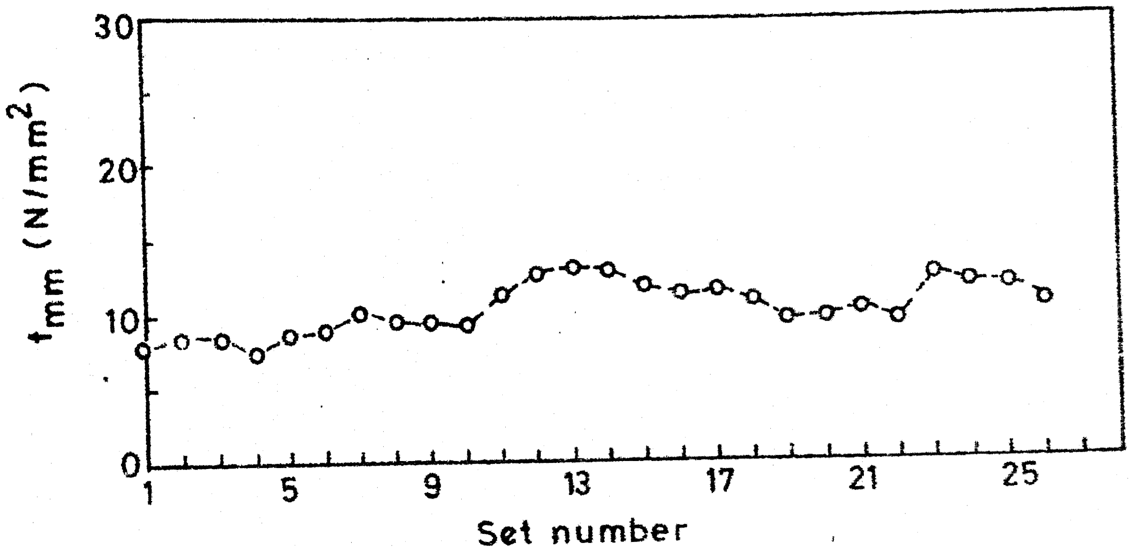


(a) Variation of mean value

Fig.2·23 Set variation of strength of mortar (1:4 mix)



(a) Variation of coefficient of variation



(b) Variation of mean value

Fig. 2·24 Set variation of strength of mortar (1:5 mix)

the three mixes are given in Table 2.16. Chi-square tests for normal and lognormal distribution is carried out for each mix. The results of chi-square test and the maximum significance level (p-level) at which the test is satisfied is given in Table 2.17. The histograms and the fitted distributions of three mixes are shown in Fig. 2.25 through Fig. 2.27.

The coefficient of variation of the strength of mortar cubes varied from 10 percent to 18 percent. Data of all the three mixes are found to be slightly positively skewed. The variation of the mortar strength in the field cubes is likely to be more by twenty percent and appears to be consistent with that of the concrete cubes. Normal distribution fits the data at 10 percent level of significance in all the cases, whereas lognormal distribution fits at 5 percent level of significance. Normal distribution appears to be a better fit over lognormal distribution.

2.4.3 Analysis of probability of failure

The characteristic strength of mortar (f_{mk}) of a particular mix for an accepted probability of failure (p_f) can be computed as

$$p_f = P\ (f_m \leq f_{mk}) \qquad (2.20)$$

where

f_{mk} = characteristic strength of mortar

and p_f = accepted risk or probability of failure.

Table 2.16: Statistical Analysis of Strength of Mortar

Mix Designation	n	Mean	δ(%)	C_s	C_k	Range	
						Lower	Upper
1:3	168	21.54	13.3	0.290	3.207	15.20	31.64
1:4	168	14.29	10.0	0.073	2.969	10.05	17.89
1:5	156	10.42	18.1	0.128	2.252	6.35	15.00

Table 2.17 : Chi-Square Test Result of Strength of Mortar

Mix Designation	Normal					Log Normal				
	f_{mm}	s	χ^2	ν	p-level (%)	m	σ_{ln}	χ^2	ν	p-level (%)
1:3	21.54	2.86	4.78	6	57.0	21.35	0.132	4.14	7	76.0
1:4	14.29	1.44	3.32	7	85.0	14.22	0.100	5.86	7	55.0
1:5	10.42	1.89	9.90	7	19.0	10.26	0.179	12.44	7	7.5

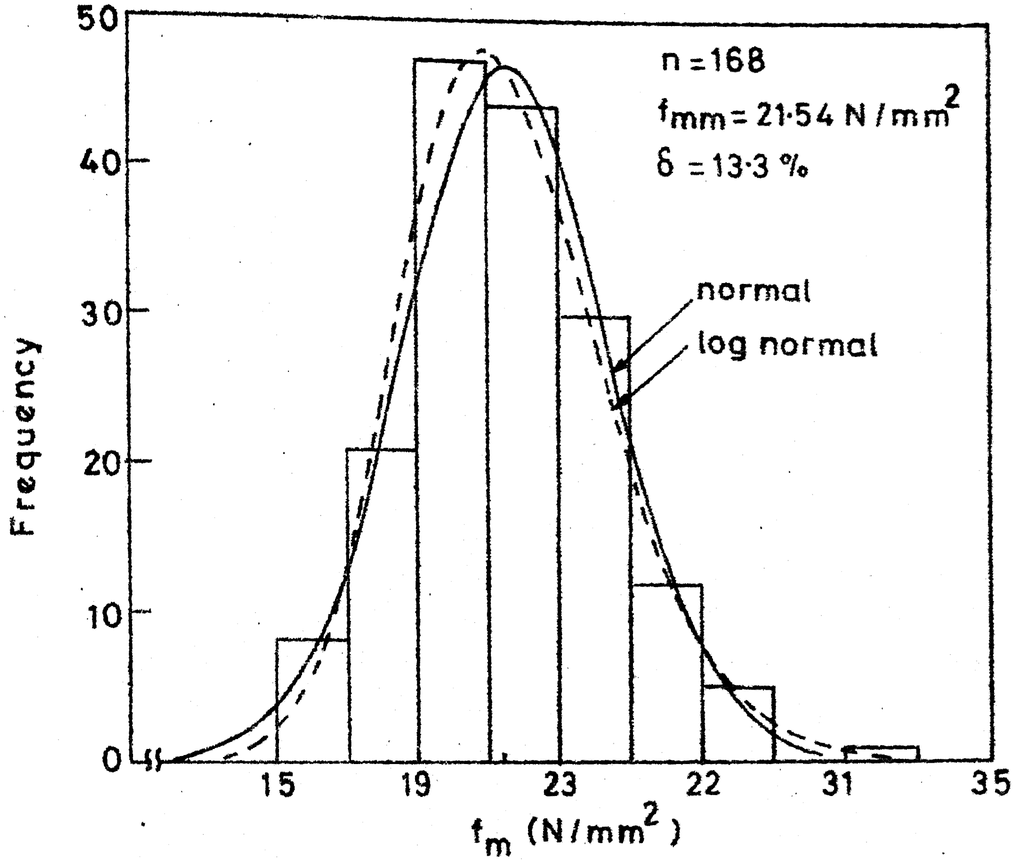


Fig. 2·25 Histogram of strength of mortar (1:3mix)

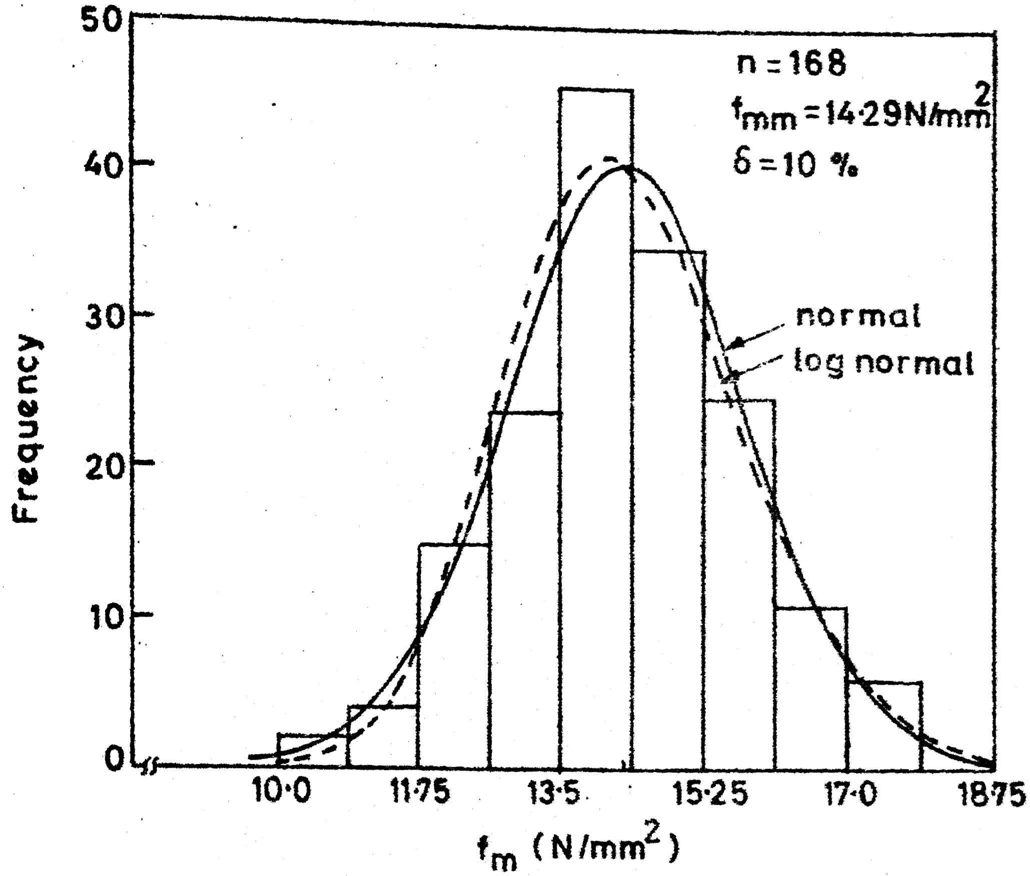


Fig. 2·26 Histogram of strength of mortar (1:4 mix)

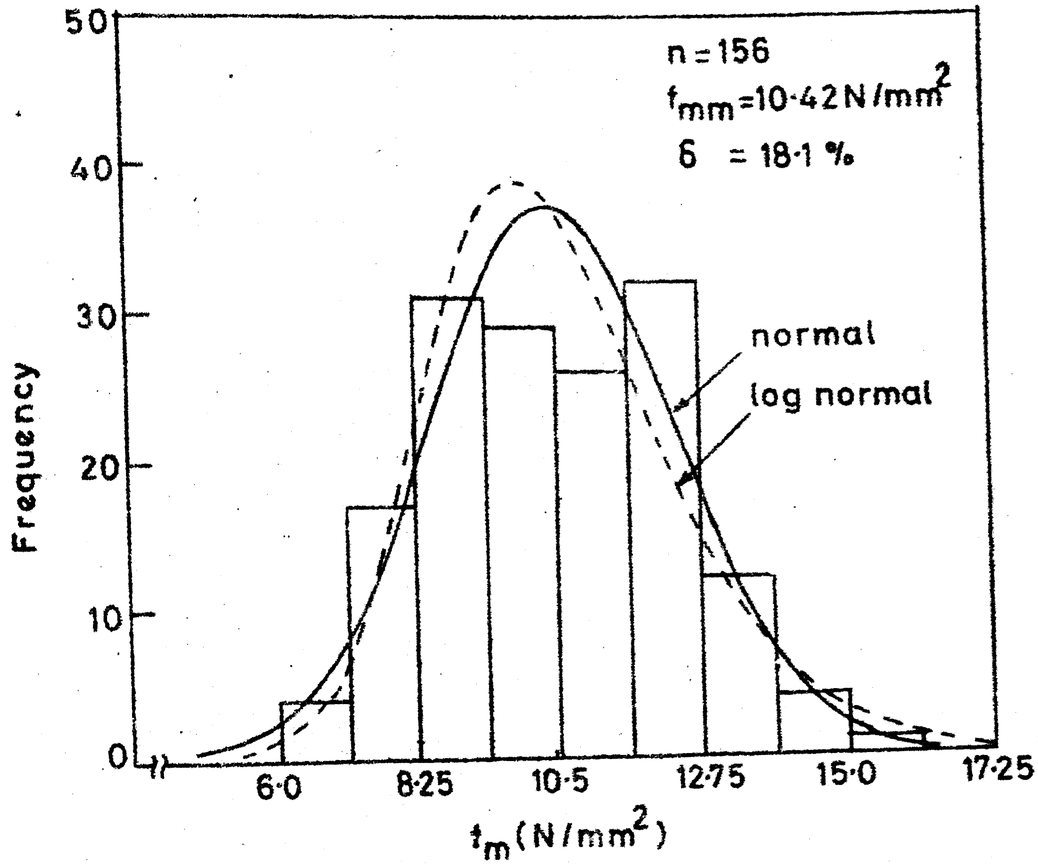


Fig. 2·27 Histogram of strength of mortar (1:5mix)

Assuming strength of mortar follows normal distribution, Eq. 2.20 can be written as

$$p_f = \phi \left(\frac{f_{mk} - f_{mm}}{s} \right) \tag{2.21}$$

or

$$f_{mk} = f_{mm} + ks \tag{2.22}$$

where f_{mm} = mean strength of mortar

s = standard deviation

k = $\phi^{-1}(p_f)$

and $\phi^{-1}(.)$ = inverse cumulative distribution function of a standardized normal random variable.

If strength of mortar follows lognormal distribution, Eq. 2.20 can be written as

$$p_f = \phi \left(\frac{\ln(f_{mk}/m)}{\sigma_{ln}} \right) \tag{2.23}$$

or

$$f_{mk} = \frac{f_{mm}}{\sqrt{(\delta^2+1)}} \exp\left[k \sqrt{(\ln(\delta^2+1))} \right] \tag{2.24}$$

where $\delta = \dfrac{s}{f_{mm}}$

$\hat{\sigma}^2_{ln} = \ln(\delta^2+1)$

and $m = f_{mm} \exp(-0.5\sigma^2_{ln})$

For an accepted probability of failure of 5 percent (i.e., $p_f = 0.05$)

$$k = \phi^{-1}(p_f) = -1.645$$

and Eqs. 2.22 and 2.24 becomes

$$f_{mk} = f_{mm} - 1.645\ s \qquad (2.25)$$

$$f_{mk} = \frac{f_{mm}}{\sqrt{(\delta^2+1)}} \exp\left[- 1.645\sqrt{(\ln(\delta^2+1))}\right] \qquad (2.26)$$

Eqs. 2.25 and 2.26 represents the relationship between characteristic strength and mean strength for normal and lognormal distribution respectively.

2.5 Statistical Analysis of Masonry Strength

2.5.1 General

Brick masonry is defined as an assembly of brick units bonded together by mortar in a predetermined orientation. The strength of brick masonry depends on several parameters. Some of the parameters are given below:

i) the strength of individual brick units

ii) the strength of mortar

iii) the thickness of mortar joint

iv) layout and orientation of bricks.

Since strength of brick and strength of mortar are themselves random variables, the strength of masonry also becomes a random variable. The variability of masonry strength depends upon several factors, some of which are listed below:

i) variability of strength of bricks

ii) variability of strength of mortar

iii) variability of joint thickness

iv) variability of workmanship.

The effect of workmanship and other variables on strength of masonry has been discussed by Hendry (36) in a great detail. To study the statistical behaviour of masonry strength, sufficient data is needed which presently is not available in India. One of the main reason is that regular testing of prisms is not carried out to ensure the quality of brickwork and workmanship even in large construction projects. Deterministic formula derived from experimental investigations (1, 2) and SCPI recommendation (13,95) are used to simulate masonry strength to study the statistical behaviour.

2.5.2 Masonry strength

From the experimental investigations undertaken at I.I.T. Kanpur, the strength of brick masonry can be obtained as (1,2),

$$f_w = k_w \sqrt{(f_b f_m)} \tag{2.27}$$

where

f_w = strength of masonry

f_b = strength of brick

f_m = strength of mortar

and k_w = a coefficient which depends on the layout of brick and joints.

Value of k_w is found to be 0.275 and 0.42 for loading perpendicular and parallel to bed joint respectively. The mean, standard deviation and coefficient of variation can be estimated (96) from Eq. 2.27 as (see Apendix A)

$$f_{wm} = k_w \sqrt{(f_{bm}\, f_{mm})} \qquad (2.28)$$

$$s_w = 0.5\, k_w\left(\frac{f_{bm}}{f_{mm}} \,.\, s_m^2 + \frac{f_{mm}}{f_{bm}} \,.s_b^2 \right)^{1/2} \qquad (2.29)$$

and

$$\delta_w = 0.5 \sqrt{(\delta_b^2 + \delta_m^2)} \qquad (2.30)$$

where

f_{wm}, f_{bm}, f_{mm} = mean value of f_w, f_b and f_m respectively

s_w, s_b, s_m = standard deviation of f_w, f_b and f_m respectively

and $\delta_w, \delta_b, \delta_m$ = coefficient of variation of f_w, f_b and f_m respectively.

In absence of test results, the strength of masonry can be obtained by the following formula (changed to SI units) as per the recommendations given by SCPI (13)

$$f_w = A\,(2.758 + B\, f_b) \qquad (2.31)$$

where

f_w = assumed compressive strength of masonry at 28 days in N/mm^2

f_b = average compressive strength of brick in N/mm^2, but not to exceed 96.53 N/mm^2

A = a coefficient equal to 2/3 without inspection and 1.0 with inspection

and B = a coefficient equal to 0.2 for type N mortar, 0.25 for type S mortar and 0.3 for type M mortar.

Type N, S and M mortar should be according to the specifications given in ASTM C270-68(95) and the compressive strength of mortar corresponding to different types are given below:

Mortar Type	Average Comp.strength f_m	B
N	750 psi (5.17 N/mm^2)	0.20
S	1800 psi (12.41 N/mm^2)	0.25
M	2500 psi (17.24 N/mm^2)	0.30

where

f_m = average laboratory compressive strength of 2 inch cube at 28 days.

An equation of the form $B = \alpha + \beta f_m$ is fitted to find the relationship between the coefficient B and compressive strength of mortar f_m. The value of α and β thus obtained is

$$\alpha = 0.155 \text{ and } \beta = 0.0082$$

The equation relating B and f_m is then given by

$$B = 0.155 + 0.0082\, f_m \tag{2.32}$$

Then Eq. 2.31 can be expressed as

$$f_w = A\,(2.758 + 0.155\, f_b + 0.0082\, f_b\, f_m) \tag{2.33}$$

The mean, standard deviation and coefficient of variation of f_w can be estimated from Eq. 2.33 by (see Appendix A)

$$f_{wm} = A(2.758 + 0.155\, f_{bm} + 0.0082\, f_{bm}\, f_{mm}) \qquad (2.34)$$

$$s_w = A\left[(0.155+0.0082\, f_{mm})^2\, s_b^2 + (0.0082\, f_{bm})^2\, s_m^2 \right]^{1/2} \qquad (2.35)$$

and $$\delta_w = \frac{s_w}{f_{wm}} \qquad (2.36)$$

2.5.3 Monte Carlo simulation

Monte Carlo simulation technique is applied to generate samples of masonry strength by the deterministic formula given in Eq. 2.27 and Eq. 2.33. One of the usual objectives in using the Monte Carlo simulation technique is to estimate certain parameters like mean, standard deviation etc. and the probability distributions of random variables whose values depends on several other random variables of specified distributions. Let Y be a random variable which is a function of several independent random variables X_j with probability density function f_{X_j}. Y is given by

$$Y = g\,(X_1,\ X_2 ----, X_j,\ ---- X_n) \qquad (2.37)$$

Monte carlo simulation is carried out in the following way(78)

i) A value X_{jk} is generated for each random variable X_j from their specified distribution, $F_{X_j}(x_j)$ or probability density $f_{X_j}(x_j)$ independently.

ii) The kth function value Y_k is calculated from the Eq. 2.37 as

$$Y_k = g\ (X_{1k}, X_{2k}, ----, X_{jk}, ----- X_{nk})$$

and stored in a array.

iii) Step (i) and (ii) are repeated for k = 1 to N (usually the required number of experiments N is decided before hand) times. The array of Y will then contain N simulated samples of Y.

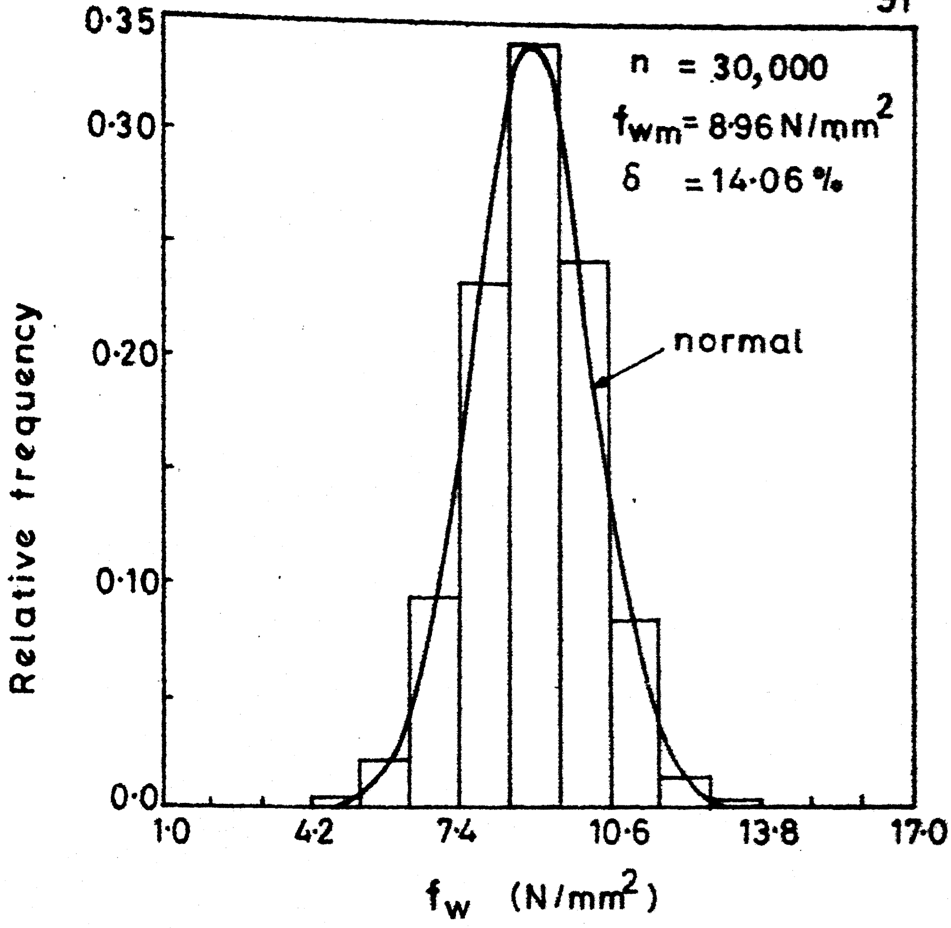
iv) The generated samples of Y is then used to compute the statistical moments and chi-square test is conducted to fit a particular distribution to the data.

Strength of brick f_b and strength of mortar f_m are assumed independent and normally distributed random variables in the Monte Carlo simulation. The mean and standard deviation of each variables are as follows

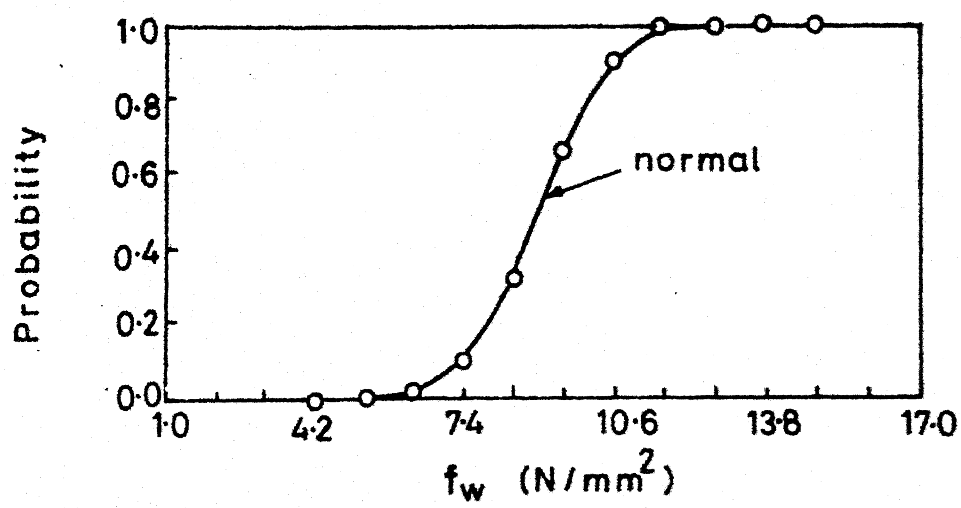
$$f_{bm} = 23.69\ \text{N/mm}^2\ ,\quad s_b = 5.489\ \text{N/mm}^2,\quad \delta_b = 0.232$$

$$f_{mm} = 21.54\ \text{N/mm}^2,\quad s_m = 2.862\ \text{N/mm}^2,\quad \delta_m = 0.133$$

30,000 samples of strength of masonry f_w are generated by simulation from the randomly generated data of f_b and f_m using their individual statistical properties as given above. Deterministic formulae given by Eqs. 2.27 and 2.33 are used. Fig. 2.28 illustrates the histogram and cumulative distribution of the simulated masonry strength using Eq. 2.27 with $k_w = 0.4$. The simulated data did not follow normal



(a) Histogram



(b) Cumulative distribution

Fig.2·28 Histogram of simulated strength of masonry using Eq.2·27

distribution even at 0.1 percent significance level. The central region of the simulated data matched well with normal distribution, but mismatching was observed at either tails.

The histogram and cumulative distribution of the simulated masonry strength using Eq. 2.33 with A = 1.0 is shown in Fig. 2.29. Simulated data based on Eq. 2.33 followed normal distribution at 0.5 percent significance level. Mean, standard deviation and coefficient of variation computed from Eqs. 2.28 to 2.30 and Eqs. 2.34 to 2.36 are given in Table 2.18. Corresponding values estimated from the simulated data are also presented in Table 2.18.

2.5.4 Characteristic strength of masonry

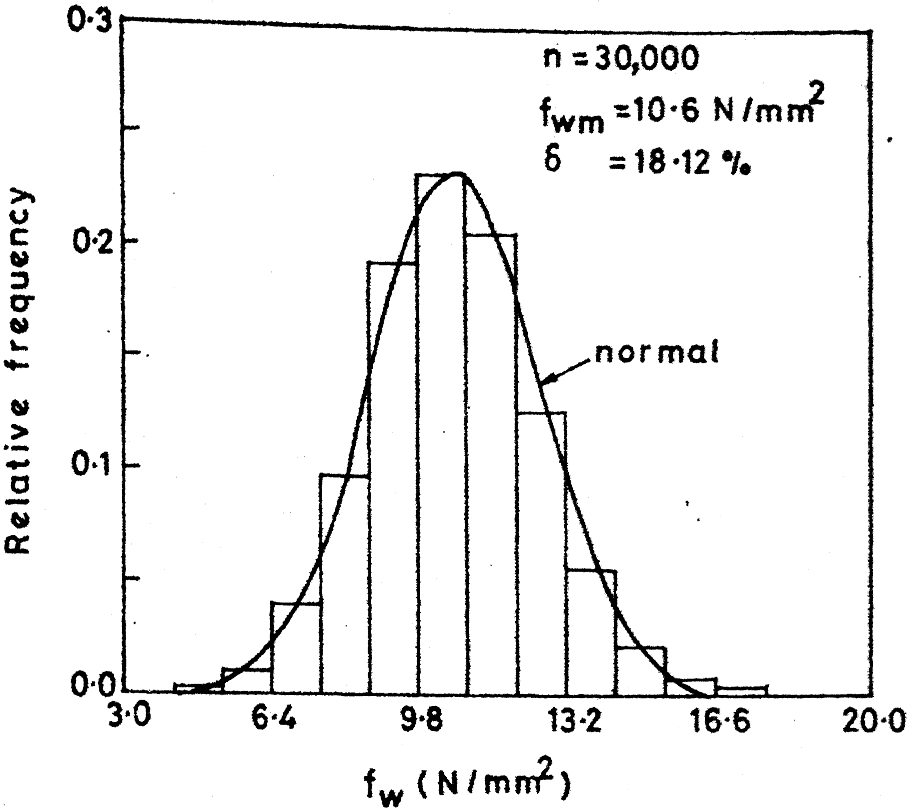
Characteristic strength of masonry can be defined as the strength under which the test results fall, with an accepted probability p_f. The relationship between characteristic strength and mean strength depends on the choice of assumed distribution. If masonry strength f_w follows normal, then the relationship is given by

$$f_{wk} = f_{wm} + k\ s_w \tag{2.38}$$

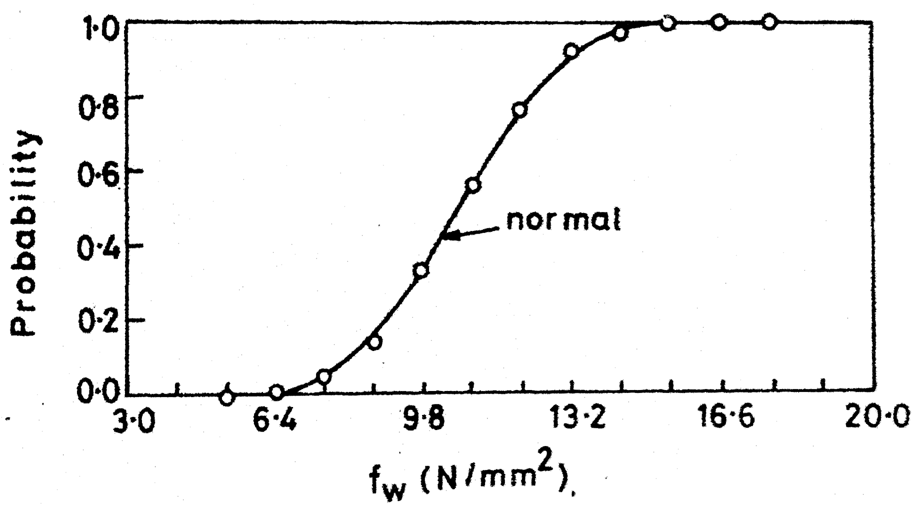
and for f_w following lognormal distribution is given by

$$f_{wk} = \frac{f_{wm}}{\sqrt{(\delta_w^2+1)}} \exp\ [k \sqrt{\ln\ (\delta_w^2+1)}\ ] \tag{2.39}$$

where $k = \phi^{-1}\ (p_f)$



(a) Histogram



(b) Cumulative distribution

Fig. 2·29 Histogram of simulated strength of masonry using Eq. 2·33

Table 2.18 : Mean, Standard Deviation and Coefficient of Variation of Strength of Masonry

Estimated Values	$f_w=k_w\sqrt{(f_b f_m)}$		$f_w=A(2.758+0.155f_m+0.0082 f_b f_m)$	
	By simulation	From Eqs. 2.28 to 2.30	By simulation	From Eqs. 2.34 to 2.36
$f_{wm}(N/mm^2)$	8.958	9.036	10.615	10.614
$s_w(N/mm^2)$	1.259	1.207	1.923	1.903
δ_w	0.1406	0.1335	0.1812	0.1793

Note:

$f_b \rightarrow N\ (23.69,\ 5.489)\ N/mm^2$, $\delta_b = 0.232$

$f_m \rightarrow N\ (21.54,\ 2.862)\ N/mm^2$, $\delta_m = 0.133$

$k_w = 0.4$ and $A = 1.0$

The above two equations are rearranged as

$$\frac{f_{wk}}{f_{wm}} = (1 + k\,\delta_w) \qquad (2.40)$$

$$\frac{f_{wk}}{f_{wm}} = \frac{1}{\sqrt{(\delta_w^2+1)}}\ \exp\left[\,k\,\sqrt{\ln(\delta_w^2+1)}\,\right] \qquad (2.41)$$

Fig. 2.30 shows the graph of the above equations for an accepted probability of 5 percent, 2.5 percent and 1 percent (p_f = 0.05 , 0.025 and 0.01) and for coefficient of variation δ_w ranging from 0 to 0.30. As the coefficient of variation increases, the ratio between characteristic to mean strength decreases. For a specified characteristic strength the required mean value predicted by normal distribution is always higher than that predicted by lognormal distribution. Thus prediction through Eq. 2.38 of normal distribution will be always on safer side as compared to lognormal distribution. BS 5628: Part 1 (43) defines characteristic strength of masonry at 5 percent risk. IS Code (41) and CEB-FIP committee (82) recommends the prediction of characteristic strength of concrete at 5 percent acceptable risk

$$f_k = f_m - 1.645\ s \qquad (2.42)$$

where f_k = characteristic strength of concrete

f_m = mean strength of concrete

s = standard deviation.

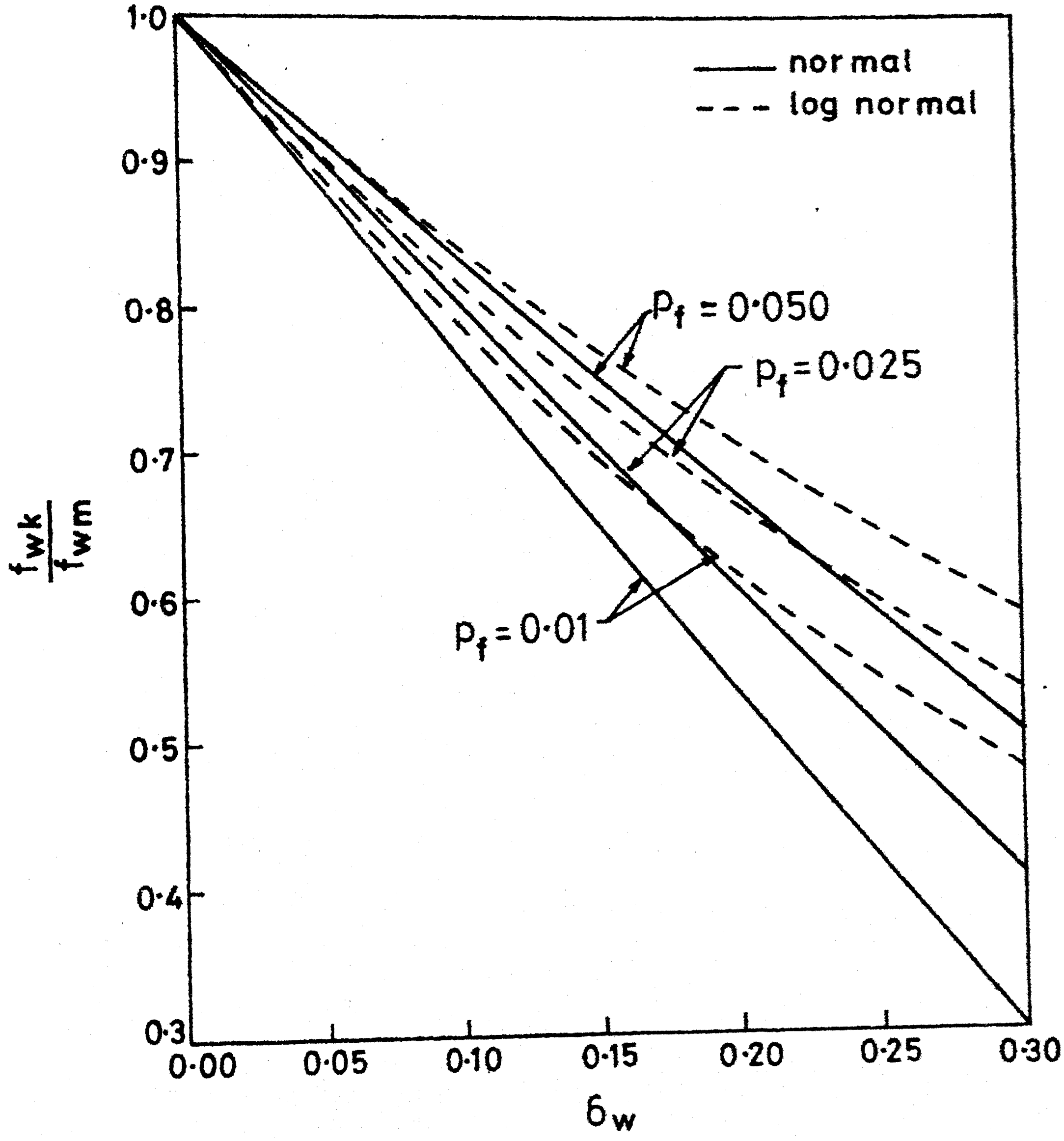


Fig. 2·30 Relation between characteristic and mean strength of masonry

2.6 Discussions and Conclusions

2.6.1 General

Brick samples were collected from different brick manufacturers of Kanpur zone and Faroke (Kerala) zone to study the quality of bricks manufactured in the country. The thickness of mortar joint from different existing buildings was measured and statistical analysis has been carried out. Cubes of three different mixes were cast and tested to study the variability of strength of mortar. Masonry strength was simulated in a digital computer through different deterministic formula and the probability behaviour of masonry strength is studied. Bricks and masonry is discussed separately in the following sections.

2.6.2 Bricks

The coefficient of variation of brick dimensions of different brick manufacturers has been found to be within 3 percent. Statistical variations of different dimensions of precast elements of prestressed concrete T beams under controlled conditions were studied by Ranganathan (85). It was reported that the coefficient of variation in width and depth of laboratory made beam sections was within 1.6 percent whereas that corresponding to the thickness and cover was in the range of 5 to 8.6 percent. Hence, it can be concluded that the consistency maintained in the

dimensions of the hand made bricks is satisfactory and can be compared well with that maintained in concrete construction.

Bricks of Kanpur zone have been found much stronger than those of Faroke. The coefficient of variation of compressive strength of bricks of different manufacturers has been found to vary from 15 to 24.6 percent. A similar statistical analysis carried out on several sets of concrete cube samples supplied to the laboratory for quality analysis, gave coefficient of variation of 23.9 and 18.8 percent for all the random sets of M15 and M20 concrete respectively (97). The corresponding values of M25 and M30 concrete were found to be 12 and 13 percent respectively. The coefficient of variation of compressive strength of brick can be compared with that of M15 concrete. The cube strength results of M15 concrete which were analysed are of the nominal mix proportioned concrete. Bricks from different manufacturers put together resulted in coefficient of variation of strength as high as 29 to 31 percent. The same degree of variability is also seen in random lot.

2.6.3 Masonry

The strength of masonry depends on several factors such as strength of individual brick unit, strength of mortar and thickness of bed joint, the type of supervision given in the construction, etc. Effect of uncertainity or

variability of different parameters are responsible for the variability of masonry strength. Assuming that no single parameter affects the variability of strength of masonry too much, the strength of masonry can be assumed as normally distributed by the central limit theorem. However, actual data of masonry strength from the field is to be collected to estimate the variability of masonry strength and its probability behaviour.

2.6.4 Conclusions and recommendations

The following conclusions and recommendations are arrived at in the present investigation:

1. Dimensions of bricks manufactured from a particular manufacturer have a very small coefficient of variation ranging from 1 to 3 percent and have tendency towards deterministic values. The dimensions of brick can be assumed to be normally distributed for practical purposes.
2. Density of dry bricks of a manufacturer follows normal distribution and the coefficient of variation is found to be 5 percent which reflects a good quality control over the mixing and compaction of soil in brick making.
3. Percent of water absorption did not fit normal or lognormal distribution and the coefficient of variation is found to be anywhere between 13 to 22 percent.

4. Bricks of Kanpur zone are much stronger than those made in some places of Faroke (Kerala).

5. Strength of brick follows normal distribution in most cases specially for a particular manufacturer.

6. The thickness of mortar joint follows normal distribution at 1 percent significance level and lognormal at 2.5 percent significance level.

7. Compressive strength of mortar (laboratory specimens) follows normal distribution at 10 percent significance level and has a coefficient of variation of 10 to 18 percent. The coefficient of variation of field cubes is expected to be about 20 percent more than that observed in the laboratory and could be anywhere between 12 to 22 percent. This is comparable with the quality of M15 concrete. Field data need be collected.

8. Actual field data on masonry strength need to be collected for a better understanding of the statistical behaviour of masonry strength. Simulated data based on deterministic formula

$$f_w = k_w \sqrt{(f_b f_m)}$$

derived from limited experimental investigation did not follow normal distribution even at 0.1 percent significance level. Simulated data of masonry strength based on the deterministic formula

$$f_w = A(2.758+0.155f_b+0.0082f_bf_m)$$

followed normal distribution at 0.5 percent significance level. Strength of masonry may be assumed as normally distributed random variable.

CHAPTER 3

RELIABILITY OF CHI-SQUARE TEST

3.1 Introduction

Load and strength of materials like concrete, masonry, reinforcement etc. have been recognised as random variables. The probability of failure of a structural member under the action of external load can be calculated as

$$p_f = P (R \leq S) \tag{3.1}$$

where R and S represent resistance and load respectively. Probability of failure p_f, is sensitive on the choice of the probability distribution of R and S. To compute p_f, the data of load S may be collected from a load survey on existing buildings and then an empirical probability distribution is to be fitted to the data. Similarly, data of resistance R is to be collected and then an empirical probability distribution model is to be fitted. After arriving at the probability distribution of the individual variable, one can easily compute the probability p_f, by Eq. 3.1 assuming R and S are independent random variables.

In most cases, only the data is available from field, but the probability distribution, along with the parameters of the distribution is unknown to the engineer or the statistician. Suppose, data of a random variable is collected

from the field and one has to fit a probability distribution model to the data. For obvious reasons, one will face the following questions:

(i) Since there are many standard probability distributions available, which distribution should be tried to fit the data?

(ii) How the parameters of the distribution could be estimated from the data and what sort of statistical tests are to be done to fit a particular probability distribution?

One has to exercise his engineering or scientific judgement to select a particular probability distribution considering the actual physical phenomena. Two or more probability distributions can also be selected as equally competent by considering the actual phenomena. For example, the strength of masonry is a random variable. The variability of masonry strength depends on several factors, which are themselves random variables. So, if the uncertainity of the individual factors are assumed to give an additive effect on the randomness of strength of masonry one can select normal distribution as a model to fit the masonry strength data through central limit theorem. On the otherhand, if one assumes that the randomness or variability of masonry strength comes from the multiplicative effect of several

individual factors, lognormal distribution can also be selected as the probability model since log-normal distribution comes from the multiplicative effect of individual random variables. Thus, normal and lognormal distributions are the two competent distributions to fit the data as far as the engineering judgement is concerned.

After arriving at the decision regarding the probability distribution one has to test whether the distribution can be accepted as a mathematical model to represent the particular random variable. If there are more than one competent distributions, decision has to be taken as to which one of them represents the data in a better way. This leads to non-parametric hypothesis testing problems. Several non-parametric methods are available in the literature for testing hypothesis. Chi-square test and Kolmogorov-Smirnov test (K-S test) are the two popular tests used in hypothesis testing, although both of them suffer from some disadvantages.

The chi-square statistic computed in the chi-square test depends on the way how the groups are formed and the number of class intervals chosen etc. Thus, from the same data, one can accept a particular null hypothesis at a particular significance level whereas another person by grouping the data into different classes may reject the null

hypothesis at that particular significance level. Some of these problems of chi-square test have been highlighted in this chapter and a practical recommendation is given for conducting this test. Kolmogorov-Smirnov test does not group the data like chi-square test and arbitary decisions like number of classes, width of class etc. are not involved in this test. Hence, from a particular data and a particular null hypothesis, the decision of acceptance or rejection of null hypothesis at a particular significance level is unique and does not depend on the person concerned. The standard tables used for K - S test are valid when the null distribution is completely specified but if one or more parameters have to be estimated from the data then the tables are no longer valid. Tables are given for the Kolmogorov-Smirnov statistic for different null distributions (such as uniform, exponential, normal) when the parameters are not specified but have to be estimated from the data. These tables are generated by Monte Carlo simulation and are presented in this chapter.

3.2 Hypothesis Testing and Significance Level

Let $X_1, X_2, .. X_n$ be the n independent observations of a random variable X with unknown probability distribution function F_X. To test whether the observations come from a population following probability distribution function F_0,

a null hypothesis H_o that the random variable follows a probability distribution F_o is set up against an alternative hypothesis H_1 that it does not follow F_o. Mathematically it can be expressed as (93,98)

$$H_o \; : \; F_X = F_o$$

$$H_1 \; : \; F_X = \text{not } F_o$$

A statistic D is then formed under the null hypothesis H_o. The null hypothesis is accepted at a particular level of significance α if the observed value of the statistic D is less than the critical value D_α. If the distribution function of D is known then the critical value D_α can be found as

$$P\,(D > D_\alpha\,) = \alpha$$

or $$D_\alpha = F_D^{-1}(1-\alpha) \tag{3.2}$$

where $F_D^{-1}(.)$ = inverse distribution function of D.

Two types of errors are involved in hypothesis testing and they are called type I and type II errors. They are as follows:

Type I : One may wrongly reject H_o, when it is true.

Type II : One may wrongly accept H_o, when it is false.

The probability of committing a type I error is called the level of significance α (also called size of the test), and the probability of committing a type II

error is a function of the alternative hypothesis H_1 and is usually denoted by β. $(1-\beta)$ is called the power of the test of the hypothesis H_o against the alternative hypothesis H_1. The complete specification of H_1 is essential, since power is a function of H_1.

Usually, a fixed value of α is chosen ($\alpha = 0.01, 0.05,$ or 0.10) before hand and the test is conducted. On the otherhand, the maximum significance level α_{max} at which the null hypothesis can be accepted, may be computed without prior choice of α. Then, the null hypothesis at any significance level less than α_{max}, can be accepted. The maximum significance level α_{max} at which the null hypothesis can be accepted is called p-level and can be computed as

$$p = \alpha_{max} = P (D > D_o) \tag{3.3}$$

where D_o is the observed value of the statistic D.

3.3 Reliability of Chi-square Test

3.3.1 Chi-square test

In the usual Pearson chi-square test, the observations are grouped into k classes and the chi-square statistic is computed as

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{3.4}$$

where

χ^2 = Chi-square statistic

k = number of classes

O_i = observed frequency of the ith class

E_i = expected frequency of the ith class corresponding to null hypothesis H_o (i.e., F_o).

The statistic χ^2 has a χ^2 - distribution with k -1 degrees of freedom as n becomes large (96). When the parameters of the null hypothesis F_o are estimated from the data by the maximum likelihood method, the above statistic is distributed as χ^2 with ν degrees of freedom, where ν is defined as

$$\nu = k - r - 1 \qquad (3.5)$$

where r is the number of parameters estimated from the data.

The chi-square method gives a numerical measure of the difference between the observed and the theoretical distribution. Let χ_o^2 denotes the observed value of χ^2. For a given significance level α , the null hypothesis is accepted if

$$\chi_o^2 \leq \chi_{\alpha,\nu}^2$$

where $\chi_{\alpha,\nu}^2$ is defined by the following equation

$$P\ (\chi^2 > \chi_{\alpha,\nu}^2) = \alpha \qquad (3.6)$$

The maximum significance level at which the null hypothesis can be accepted is computed as

$$p = \alpha_{max} = P\ (\chi^2 > \chi_o^2) \qquad (3.7)$$

The computed value of chi-square statistic depends on the number and width of classes. Care has to be taken that $E_i \geq 3$ for all classes.

3.3.2 Choice of class interval

The way the data is grouped to form the classes plays an important role in the chi-square test. Two schemes are generally adopted in practice for the choice of class intervals:

(i) equal length of interval

(ii) equi-probable cells

Struges (99) suggested a formula for the choice of class interval which can be computed as

$$C = \frac{R}{1 + 3.222 \log_{10} n} \qquad (3.8)$$

where

C = class interval

R = the range of the observations

and n = number of observations.

Benjamin and Cornell (93) also recommended the same formula given by Struges (99) as a practical guide to choose the number of classes (k) for chi-square test and is given by

$$k = 1 + 3.2 \log_{10} n \qquad (3.9)$$

Mann and Wald (100) proposed a formula for the optimal choice of the number of classes for chi-square test with equi-probable cells where the population parameters are not estimated from the data but are based on standard theory or past experience. The number of classes can be calculated as

$$k = [4 \sqrt[5]{\frac{2(n-1)^2}{c^2}}] \qquad (3.10)$$

where c is determined such that

$$\frac{1}{\sqrt{2\pi}} \int_{c}^{\infty} e^{-x^2/2} dx = \alpha \qquad (3.11)$$

where α is the level of significance.

The usual chi-square test is unreliable (101) for a continuous variate when equal width classes are formed, since it involves three arbitary decisions such as starting point, width and number of classes. Gumbell (101) has shown through examples that from the same observations, different statisticians, equally well trained and equally careful may obtain different probabilities of the critical region (i.e. different p-level) and may proclaim anyone of these as final. Thus, chi-square test does not lead to a decision whether a hypothesis has to be rejected or not. When equi-probable cells are used in the chi-square test, the decision about the starting point and width of classes

does not arise. Williams (102) suggested that the number of classes given by Mann and Wald (100) may be halved for practical purposes without greatly affecting the power of the test.

Watson (103) recommended that at least 10 classes of not very unequal probability content should be used. Hamdan (104) showed that a number of classes between 10 to 20 is adequate to ensure a reasonably sensitive test, and the optimum choice of class boundaries corresponds to equal class width of about 0.4 standard deviation being slightly more powerful than the equal class probability partition. The paper gives a direct approach to the problem of boundary determination in the case of the chi-square test for location of the normal distribution. Dahiya et al.(105) discussed about the number of classes to be used in testing for normality against certain families of alternatives. They recommended a range of choice of k for several different alternatives in testing for normality.

3.3.3 Variation of significance level

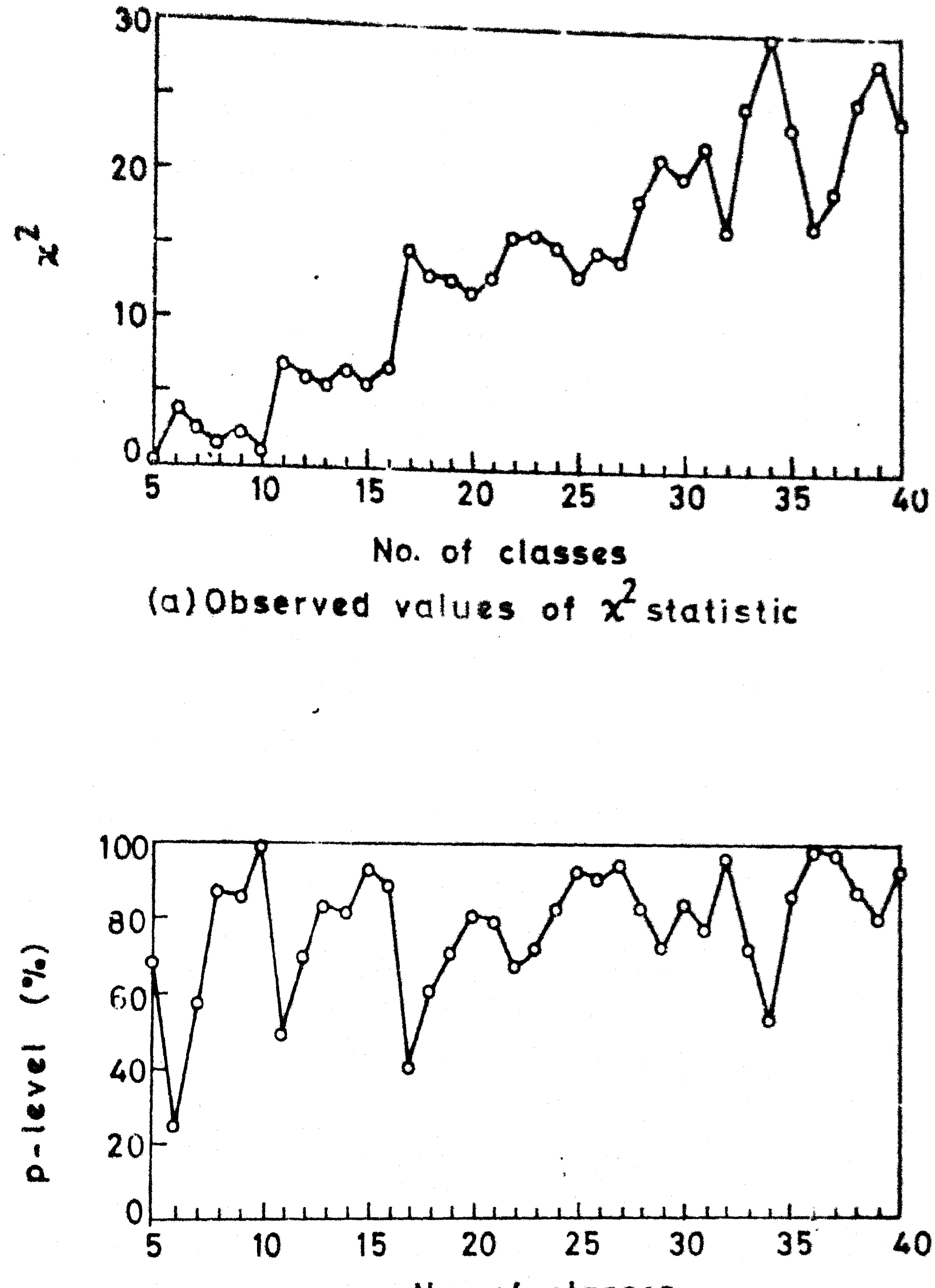
Chi-square test when used with equal width classes has some limitations as it depends upon three arbitrary decisions discussed earlier. If a certain choice of the intervals gives a good fit, it cannot be concluded that a broader or narrower classification gives the same or better

fit. It may so happen that a different classification may lead to rejection of the null hypothesis. Observed value of Chi-square may change considerably through different sets of starting point, although number of degrees of freedom remains constant. The arbitrary choices of the three variables can be made to a single variable, if cells of equal probability are used instead of cells of equal width. Best choice of the number of classes k, given by Mann and Wald (100) is valid for small levels of significance and for large number of observations, but the question of the best choice of k for small number of observations and large levels of significance is not yet solved. Number of classes needed for Chi-square test as computed from Eqs. 3.9 and 3.10 for different sample sizes are shown in Table 3.1.

Data of strength of mortar of three different mixes are taken up for numerical experimentation to study the effect of number of classes on the chi-square test result. The chi-square test is carried out using cells of equal probability for different number of classes. Number of classes considered here are 5 to 40. A typical variation of chi-square statistic with number of classes for the data of 1:3 mix is shown in Fig. 3.1(a), when the null hypothesis is normal distribution, and the corresponding p-levels at which the normal distribution can be accepted is shown in Fig. 3.1(b). It can be seen from Fig. 3.1(a) that there

Table 3.1 : Number of Classes for the Chi-square Test

n	From Eq.3.9	From Eq. 3.10		
		Level of significance		
		0.01	0.05	0.10
50	7	16	18	20
100	8	21	24	26
150	8	24	28	31
200	9	27	31	35
500	10	40	45	50
1000	11	52	60	66
2000	12	69	79	87
5000	13	99	114	126
10000	14	131	150	166



(a) Observed values of χ^2 statistic

(b) Maximum significance level

Fig. 3·1 Variation of chi-square and p-level with number of classes for normal distribution

is no steady increase or decrease of chi-square value with the increase in number of classes. A random fluctuation in p-level is observed. The p-level varied from 26 to 99 percent as can be seen from Fig. 3.1(b). Similar fluctuations in p-levels are found in other two sets of data (1:4 and 1:5 mix). Mean, standard deviation, coefficient of variation and range of p-level for three sets of data are given in Table 3.2.

The maximum significance level at which null hypothesis can be accepted is not only a function of the data but also a function of the number of classes chosen. Sometimes a drastic change in p-level between any two consecutive number of classes is also observed. For example, if k=16 is used, the p-level becomes 93.5 % which signifies a very good fit of normal distribution to 1:3 mix data . On the other hand, if k =17 is used, the p-level becomes 64.5 %. Similar variations are observed in all the cases which clearly show that the test should not be done by just taking a particular value of k and come to a final decision about the acceptance or rejection of the null hypothesis.

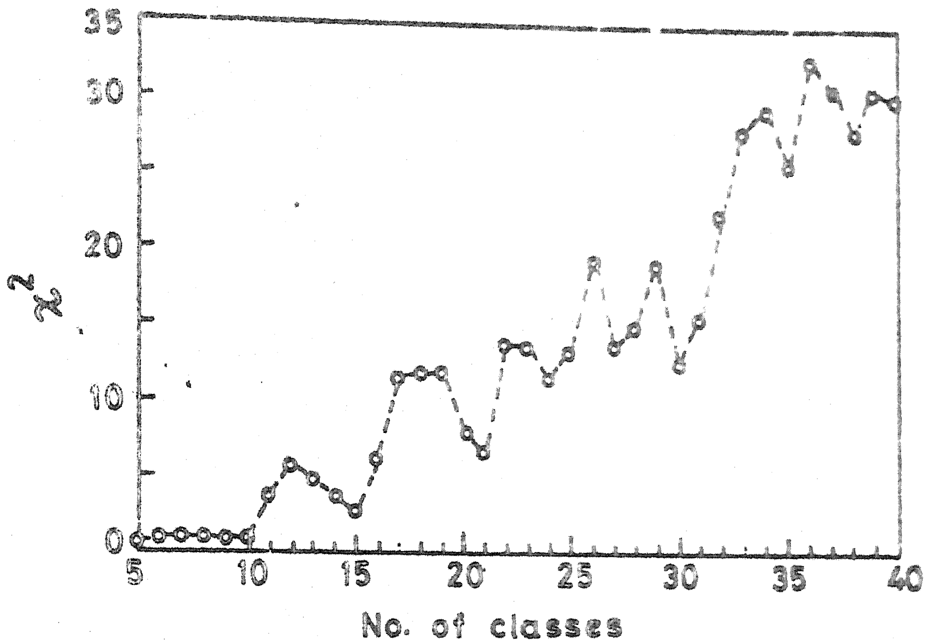
If the null hypothesis is assumed to be lognormal, the value of χ^2 also changes with the choice of number of classes. The variation of χ^2 and p-level with the number of classes for 1:3 mix data is shown in Fig. 3.2. The p-level varied from 46.7 % to 99.7 % with a mean value of 82.8 %. Mean, standard deviation, coefficient of variation and range of p-level for three different mixes are given in Table 3.3.

Table 3.2 : p-Level (%) for Normal Distribution

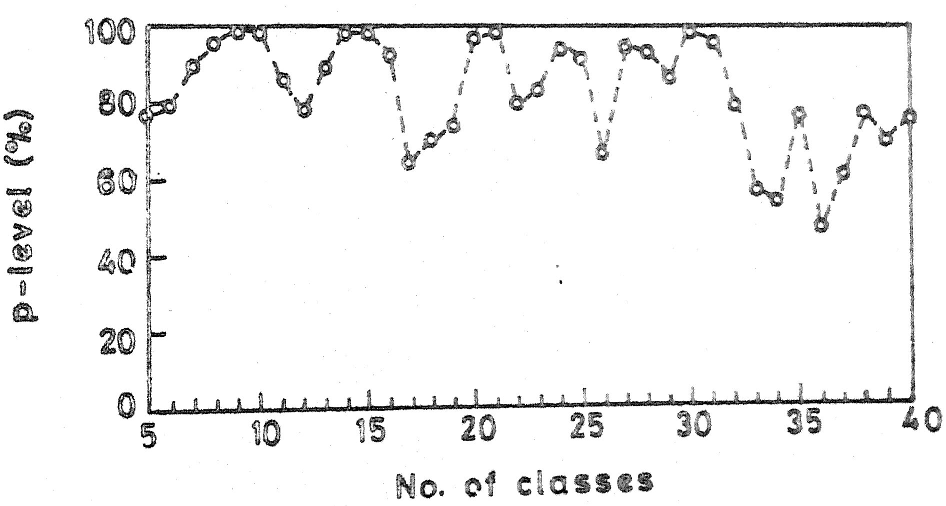
Sample designation	Mean	S.D.	C.O.V.	Range	
				Lower	Upper
1:3 mix	78.316	16.746	0.2138	26.15	98.96
1:4 mix	21.608	17.987	0.8324	0.92	67.76
1:5 mix	11.302	12.467	1.1031	0.60	42.89

Table 3.3 : p-Levels (%) for Lognormal Distribution

Sample designation	Mean	S.D.	C.O.V.	Range	
				Lower	Upper
1:3 mix	82.839	14.584	0.1760	46.73	99.72
1:4 mix	30.702	22.583	0.7356	0.68	75.54
1:5 mix	8.898	10.621	1.1936	0.25	56.32



(a) Observed values of χ^2 statistic



(b) Maximum significance level

Fig.3.2 Variation of chi-square and p-level with number of classes for lognormal distribution

3.3.4 Characteristic significance level

As discussed earlier, it is found that the maximum significance level varies randomly with number of classes used in chi-square test. Choice of different value of k can lead to random results. In order to overcome this difficulty, a series of chi-square tests with different values of k may be conducted to decide the most reliable p-level.

The p-levels observed for different values of k for different mixes did not follow any particular distribution, although a certain grouping tendency has been observed. Let p_k be defined as characteristic p-level such that the probability of p-level falling below p_k is p_f^*. Assuming p follows normal distribution, the characteristic p-level p_k, can be computed as

$$p_k = p_m + k_1 s \tag{3.12}$$

where p_m = mean of the p-levels

s = standard deviation of the p-levels

$k_1 = \phi^{-1}(p_f^*)$

and $\phi^{-1}(\cdot)$ is the inverse of the cumulative standardized normal distribution function.

If the probability of falling below p_k is accepted to

be 5 percent, the characteristic p-level can be computed as

$$p_k = p_m - 1.645\ s = p_m\ (1-1.645\ \delta) \qquad (3.13)$$

where δ = coefficient of variation

$$k_1 = \phi^{-1}(0.05) = -1.645$$

Coefficient of variation of p-level is found to be as high as 120 % in some cases as shown in Tables 3.2 and 3.3. Characteristic p-level computed from Eq. 3.13 will become negative for any value of δ greater than 60.79 percent.

Theoretically, p-level cannot be negative and at the most can vary within 0 to 100 %. The theoretical or conceptual difficulty of p_k being negative can be avoided it lognormal distribution is assumed. It should be noted that the limiting value of p-level is 100 % whereas lognormal distribution permits value of p-level greater than 100 %. However, lognormal distribution can still be used to compute characteristic p-level for practical purposes. Prescribing lognormal distribution for p-level, the characteristic p-level can be computed as:

$$p_k = \frac{p_m}{\sqrt{(1+\delta^2)}} \exp[\ k_1 \sqrt{(\ln(1+\delta^2))}\ ] \qquad (3.14)$$

where δ = coefficient of variation

$$k_1 = \phi^{-1}(p_f^*)$$

For an accepted probability of 5 %, Eq. 3.14 becomes

$$p_k = \frac{p_m}{\sqrt{(1+\delta^2)}} \exp\ [\ -1.645 \sqrt{(\ln(1+\delta^2))}] \qquad (3.15)$$

The characteristic p-level defined by Eq. 3.15 reflects the random variation of p-level through mean and coefficient of variation. The characteristic p-levels computed by Eq. 3.15 for the three sets of data are given in Table 3.4 for normal and lognormal distributions.

3.3.5. Acceptance of the better fitted distribution

When there are more than one competent distribution, the problem often encountered is how to judge which distribution is a better fit to the data. For example, from engineering judgement both normal and lognormal distributions can be taken as the representative model of strength of mortar. It has to be decided which model is better.

For the same number of classes, χ^2 values can be computed for normal and lognormal distributions and the distribution which gives a less value of chi-square can be taken as a better fit compared to the other competent distribution. The computed value of χ^2 is found to be 22.05 and 15.8 for normal and lognormal distributions from Figs. 3.1 and 3.2, when the number of classes k = 31 is used. This leads to a conclusion that lognormal distribution is a better fit. On the other hand, values of χ^2 are 16.4 and 22.5 for normal and lognormal distributions respectively for k = 32, which leads to the opposite conclusion that normal distribution is a better fit. Thus, direct comparison

Table 3.4 : Characteristic p-level (%)

Sample designation	Null hypothesis					
	Normal			Lognormal		
	p_m	s	p_k	p_m	s	p_k
1:3 mix	78.316	16.746	54.09	82.839	14.584	61.21
1:4 mix	21.608	17.987	5.03	30.702	22.583	8.38
1:5 mix	11.302	12.467	1.75	8.898	10.621	1.22

of chi-square at a particular chosen number of classes may not lead to a proper conclusion. The p-levels for normal and lognormal distributions for different values of k in the case of data of 1:3 mix are shown in Fig. 3.3(a). The firm line shows the variation of p-levels for normal distribution whereas the dotted line corresponds to lognormal distribution. The p-level of lognormal distribution is higher than that of normal distribution , for k = 5 to 31. Thus, if the number of classes is anywhere between 5 and 31, lognormal distribution can be claimed to be a better fit. On the other hand, if the test is carried out with number of classes anywhere between 32 to 40, normal distribution appears to be better fit over lognormal distribution. Here , two distinct zones are observed. For k varying from 5 to 31, p-levels for normal distribution are always higher than those of lognormal and for k varying from 32 to 40, p-levels for normal distribution are always lower than those of lognormal distribution as seen in Fig. 3.3(a).

In case of data of 1:4 mix, no such distinct zones are observed. The variation of p-level for normal and lognormal distributions is shown in Fig. 3.3(b). The p-levels of normal and lognormal fit are 7.5% and 26% for k = 10, which leads to the acceptance of lognormal distribution as a better fit. But, for k = 11, p-levels are 26 % and 16 % for normal and lognormal distributions

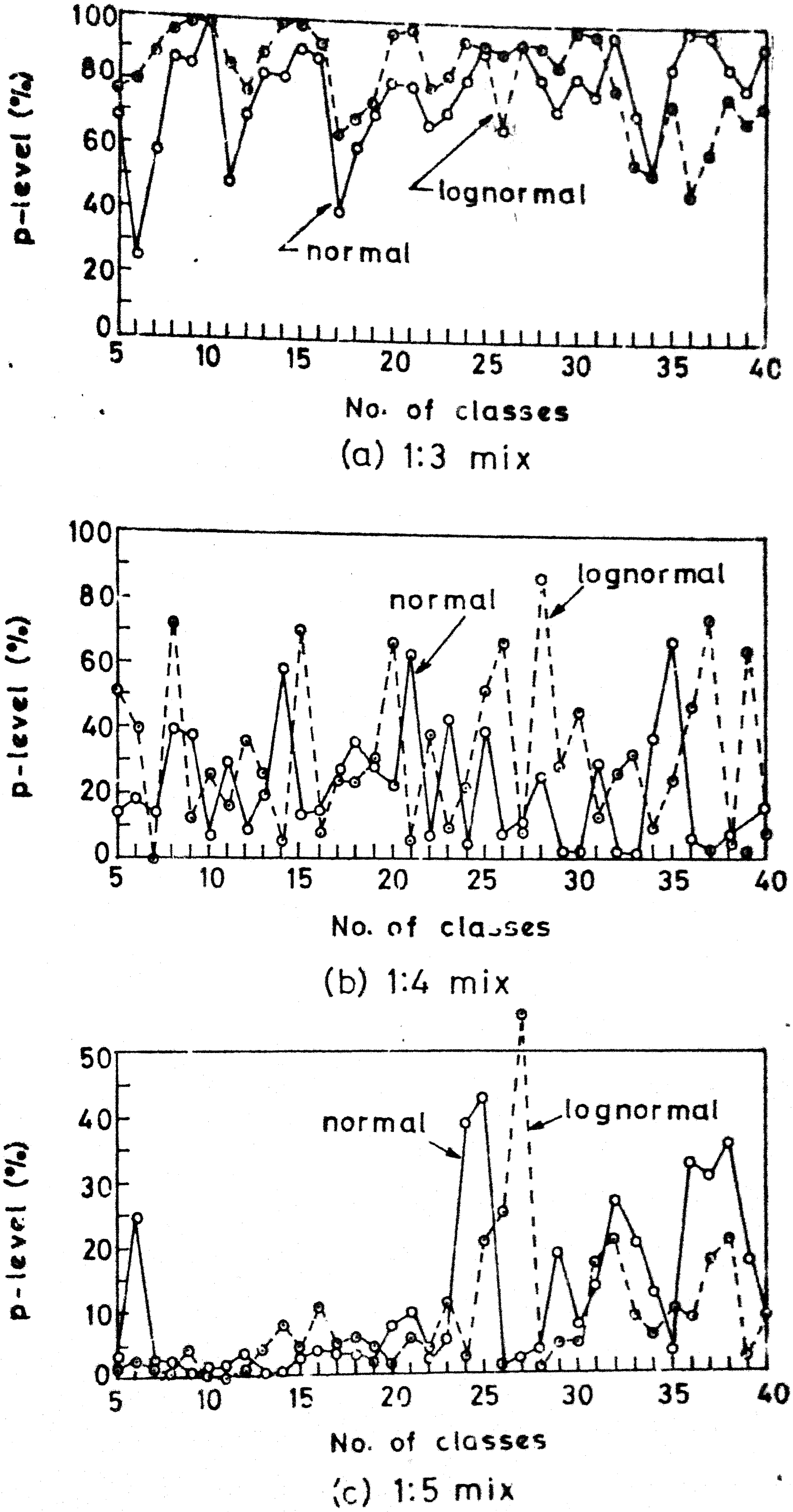


Fig.3.3 Variation of p-level with number of classes for normal and lognormal

respectively, which leads to the acceptance of normal as better fit. A change in the choice of k from 10 to 11 has changed the decision about the acceptance of the model. Similarly, k = 12 will again change the decision which has been formed by taking k = 11. The decision about the better fit of a particular distribution keeps on changing randomly with the choice of number of classes k as can be seen from Fig. 3.3(b).

Data of 1:5 mix has shown a similar type of behaviour. The variation of p-level with number of classes is shown in Fig. 3.3(c). In some regions, normal distribution appears a better fit as compared to the lognormal distribution. The variation of p-level is small in the range of k varying from 7 to 23, but p-level is found to vary drastically for k varying from 23 to 40.

In the three typical cases discussed above, it has been observed that the p-level for a particular distribution depends on the particular choice of the value of k and the decision that a particular distribution is better than the other distribution also depends on the choice of k. Thus, acceptance of a particular distribution by chi-square test is highly dependent on the choice of k.

A qualitative analysis of the reliability of chi-square test has been carried out in the present investigation.

It is clear that one should not take a particular value of number of classes and carryout the chi-square test to decide which one of the two competing distributions is a better fit. To judge which distribution is a better fit, a practical method is described as follows. First, the characteristic p-levels for the two competing distributions are to be found out after performing a series of chi-square tests with different values of k and then the distribution which gives a higher characteristic p-level should be accepted as a better representative model. Comparing the characteristic p-levels for normal and lognormal distributions given in Table 3.4 , it can be concluded that lognormal distribution is better fit for the data of 1:3 and 1:4 mixes, whereas normal distribution is better for the data of 1:5 mix.

3.4 Kolmogorov-Smirnov Test

Kolmogorov-Smirnov test (K-S test) is an alternative to chi-square test. In this test, a statistic D,defined by the following formula,is computed to test whether the sample belongs to a completely specified distribution;

$$D = \max_{x} \left| F_o(x) - S_n(x) \right| \qquad (3.16)$$

where, $F_o(x)$ is the completely specified continuous cumulative distribution function and $S_n(x)$ is the sample cumulative distribution function. The statistic D is a

distribution free statistic and depends only on the sample size n (106,107). Although both chi-square and K-S tests suffer from some disadvantages, K-S test has the following major advantages over the usual chi-square test(93,96,98,107):

(a) for small sample sizes, the validity of chi-square test becomes questionable whereas K-S test can be used with small sample size,

(b) often it appears to be a more powerful test than the chi-square test for any sample size,

(c) data is not grouped into classes and hence choice of class intervals etc., does not complicate the procedure,

(d) this test considers the effect of all individual data points in the sample where as the individual effect of every data point is lost by grouping the data into classes in chi-square test.

Smirnov (106) has given a table of critical values of statistic D for use with K-S test. Massey (107) has given a table of the statistic D for different sample sizes n = 1 to 35 for levels of significance 1,5,10,15 and 20 percent. A more extensive table was given by Miller(108).

All these tables are valid only when the null distribution is completely specified. When the parameters have to be estimated from the sample, then the statistic D

is no longer a distribution free statistic and the tables are no longer valid. If the test is used with the parameters estimated from the sample, the critical values of D are to be decreased than those given in the standard tables(106,107, 108) as suggested by Massey (107). Lilliefors (109) has given a table to be used with the Kolmogorov-Smirnov statistic for testing whether a set of observations is from a normal population when the mean and the standard deviation (obtained by n-1 as the denominator) are estimated from the data. The table was obtained by Monte Carlo simulation and is given for 1, 5, 10, 15 and 20 percent significance levels. In the present work, similar tables are generated by Monte Carlo simulation for the following distributions for all significance levels using similar procedure as given by Lilliefors(109) for normal distribution:

(i) uniform distribution

(ii) normal distribution

(iii) exponential distribution.

The procedure is illustrated in the following steps:

(i) Generate n number of values from a particular distribution F(x), with some values assigned to the parameters.

(ii) Estimate the parameters from the sample of size n.

(iii) Compute the statistic

$$D = \max_{x} |F^*(x) - S_n(x)|$$

where, $F^*(x)$ is the cumulative distribution function F(x) with parameters estimated from the data and $S_n(x)$ is the sample distribution function.

(iv) Repeat steps (i) to (iii) N times (1000 or more) and store the values of D in an array.

(v) The distribution of the statistic D is then estimated and the critical values D_α for different significance level are picked up such that

$$P\ (D > D_\alpha) \ = \ \alpha.$$

Uniform distribution:

The parameters of uniform distribution for every sample are taken as the leftmost and rightmost values respectively. The critical values of D are given in Table 3.5 for different sample size.

Normal distribution:

The parameters of the normal distribution are estimated as $\mu = X_m$ and $\sigma = s$, standard deviation with denominator n-1. The critical values of D for different sample size are given in Table 3.6. These values match well with the values given by Lilliefors(109), although there are differences observed in the third decimal place.

Exponential distribution:

The parameter of exponential distribution is estimated from the mean of the sample. The critical values of D for exponential distribution are given in Table 3.7.

Table 3.5 : Critical Values of D for Uniform Distribution

Level of significance α	Sample size n						
	4	6	10	20	25	30	over 30
0.01	0.669	0.572	0.454	0.339	0.301	0.278	$1.522/\sqrt{n}$
0.05	0.584	0.487	0.403	0.285	0.260	0.240	$1.281/\sqrt{n}$
0.10	0.532	0.449	0.368	0.259	0.235	0.216	$1.178/\sqrt{n}$
0.15	0.496	0.427	0.339	0.244	0.221	0.202	$1.104/\sqrt{n}$
0.20	0.483	0.407	0.324	0.232	0.209	0.192	$1.049/\sqrt{n}$
0.25	0.471	0.387	0.309	0.220	0.200	0.184	$1.007/\sqrt{n}$
0.30	0.459	0.368	0.294	0.210	0.194	0.175	$0.955/\sqrt{n}$
0.35	0.442	0.348	0.280	0.200	0.186	0.169	$0.916/\sqrt{n}$
0.40	0.425	0.332	0.270	0.192	0.177	0.162	$0.876/\sqrt{n}$
0.45	0.414	0.324	0.262	0.185	0.169	0.156	$0.842/\sqrt{n}$
0.50	0.395	0.317	0.255	0.178	0.163	0.152	$0.807/\sqrt{n}$
0.55	0.376	0.309	0.244	0.171	0.157	0.147	$0.779/\sqrt{n}$
0.60	0.363	0.301	0.235	0.164	0.152	0.141	$0.751/\sqrt{n}$
0.65	0.347	0.292	0.227	0.160	0.146	0.135	$0.724/\sqrt{n}$
0.70	0.331	0.279	0.218	0.153	0.141	0.130	$0.696/\sqrt{n}$
0.75	0.315	0.268	0.209	0.146	0.135	0.125	$0.671/\sqrt{n}$
0.80	0.290	0.256	0.197	0.141	0.128	0.118	$0.635/\sqrt{n}$
0.85	0.265	0.240	0.185	0.132	0.118	0.110	$0.594/\sqrt{n}$
0.90	0.250	0.219	0.175	0.122	0.110	0.100	$0.547/\sqrt{n}$
0.95	0.250	0.196	0.158	0.111	0.102	0.091	$0.489/\sqrt{n}$
0.99	0.250	0.167	0.126	0.095	0.087	0.080	$0.386/\sqrt{n}$

Table 3.6 : Critical Values of D for Normal Distribution

Level of significance α	Sample size n						
	4	6	10	20	25	30	over 30
0.01	0.415	0.374	0.299	0.226	0.201	0.187	$1.017/\sqrt{n}$
0.05	0.376	0.326	0.261	0.194	0.172	0.157	$0.890/\sqrt{n}$
0.10	0.351	0.299	0.238	0.176	0.160	0.147	$0.807/\sqrt{n}$
0.15	0.326	0.282	0.226	0.168	0.151	0.138	$0.748/\sqrt{n}$
0.20	0.307	0.268	0.216	0.161	0.143	0.131	$0.721/\sqrt{n}$
0.25	0.294	0.259	0.206	0.153	0.137	0.126	$0.687/\sqrt{n}$
0.30	0.285	0.249	0.200	0.146	0.132	0.120	$0.656/\sqrt{n}$
0.35	0.277	0.240	0.195	0.142	0.128	0.117	$0.635/\sqrt{n}$
0.40	0.271	0.233	0.188	0.138	0.123	0.113	$0.618/\sqrt{n}$
0.45	0.263	0.225	0.181	0.134	0.120	0.110	$0.599/\sqrt{n}$
0.50	0.258	0.217	0.175	0.130	0.116	0.107	$0.580/\sqrt{n}$
0.55	0.252	0.210	0.171	0.126	0.113	0.103	$0.565/\sqrt{n}$
0.60	0.245	0.204	0.164	0.122	0.109	0.100	$0.551/\sqrt{n}$
0.65	0.239	0.197	0.158	0.118	0.106	0.097	$0.536/\sqrt{n}$
0.70	0.230	0.191	0.154	0.114	0.103	0.094	$0.514/\sqrt{n}$
0.75	0.222	0.186	0.149	0.110	0.099	0.091	$0.501/\sqrt{n}$
0.80	0.212	0.178	0.143	0.107	0.094	0.088	$0.482/\sqrt{n}$
0.85	0.200	0.169	0.133	0.102	0.090	0.084	$0.458/\sqrt{n}$
0.90	0.187	0.163	0.127	0.096	0.086	0.080	$0.434/\sqrt{n}$
0.95	0.168	0.150	0.119	0.089	0.079	0.073	$0.395/\sqrt{n}$
0.99	0.144	0.126	0.103	0.076	0.068	0.062	$0.348/\sqrt{n}$

Table 3.7 : Critical Values of D for Exponential Distribution

Level of significance α	Sample size n 4	6	10	20	25	30	over 30
0.01	0.548	0.471	0.389	0.273	0.252	0.229	$1.240/\sqrt{n}$
0.05	0.482	0.406	0.322	0.231	0.212	0.195	$1.071/\sqrt{n}$
0.10	0.441	0.377	0.297	0.213	0.191	0.178	$0.979/\sqrt{n}$
0.15	0.418	0.353	0.279	0.200	0.180	0.167	$0.913/\sqrt{n}$
0.20	0.402	0.334	0.263	0.187	0.170	0.157	$0.863/\sqrt{n}$
0.25	0.386	0.318	0.252	0.177	0.163	0.150	$0.832/\sqrt{n}$
0.30	0.371	0.303	0.241	0.171	0.155	0.144	$0.801/\sqrt{n}$
0.35	0.355	0.294	0.234	0.166	0.150	0.140	$0.769/\sqrt{n}$
0.40	0.340	0.283	0.225	0.159	0.145	0.135	$0.743/\sqrt{n}$
0.45	0.327	0.276	0.216	0.154	0.140	0.130	$0.717/\sqrt{n}$
0.50	0.313	0.265	0.209	0.150	0.136	0.126	$0.689/\sqrt{n}$
0.55	0.303	0.256	0.201	0.145	0.133	0.122	$0.665/\sqrt{n}$
0.60	0.295	0.249	0.195	0.141	0.128	0.118	$0.645/\sqrt{n}$
0.65	0.286	0.241	0.189	0.136	0.123	0.114	$0.619/\sqrt{n}$
0.70	0.279	0.231	0.181	0.132	0.119	0.109	$0.596/\sqrt{n}$
0.75	0.267	0.222	0.175	0.128	0.115	0.105	$0.574/\sqrt{n}$
0.80	0.258	0.213	0.166	0.123	0.110	0.100	$0.546/\sqrt{n}$
0.85	0.244	0.206	0.156	0.116	0.105	0.095	$0.516/\sqrt{n}$
0.90	0.230	0.194	0.146	0.109	0.099	0.089	$0.490/\sqrt{n}$
0.95	0.207	0.174	0.134	0.099	0.090	0.081	$0.444/\sqrt{n}$
0.99	0.172	0.149	0.113	0.085	0.071	0.071	$0.380/\sqrt{n}$

It is observed that the distribution of D is dependent on the choice of the null distribution when the parameters are estimated from the sample itself, but independent of the numerical values of the parameters when the parameters are location or scale parameters of the distribution. The distribution of D with different levels of significance for the three null distributions is shown in Fig. 3.4. These curves are drawn for $n = 40$, since for $n > 30$, $\sqrt{n}\,D$ is found to be almost stable. The critical values obtained from Smirnov's Table(106) when the null distribution is completely specified, is also shown in Fig. 3.4. It can be noticed that the nature of the curve changes with the type of null distribution when the parameters are estimated from the data and the D values thus obtained are always smaller than those given by Smirnov(106). When the parameters estimated from the data are other than parameters of location or scale, the distribution of D is found to be a function of the numerical values of the parameters as well as the type of the distribution. This phenomenon is observed for some of the distributions such as Weibull, beta, gamma etc.

The critical values given for normal distribution can also be used for lognormal distribution. After taking natural logarithm of the sample data points, the K-S test can be performed for normal distribution.

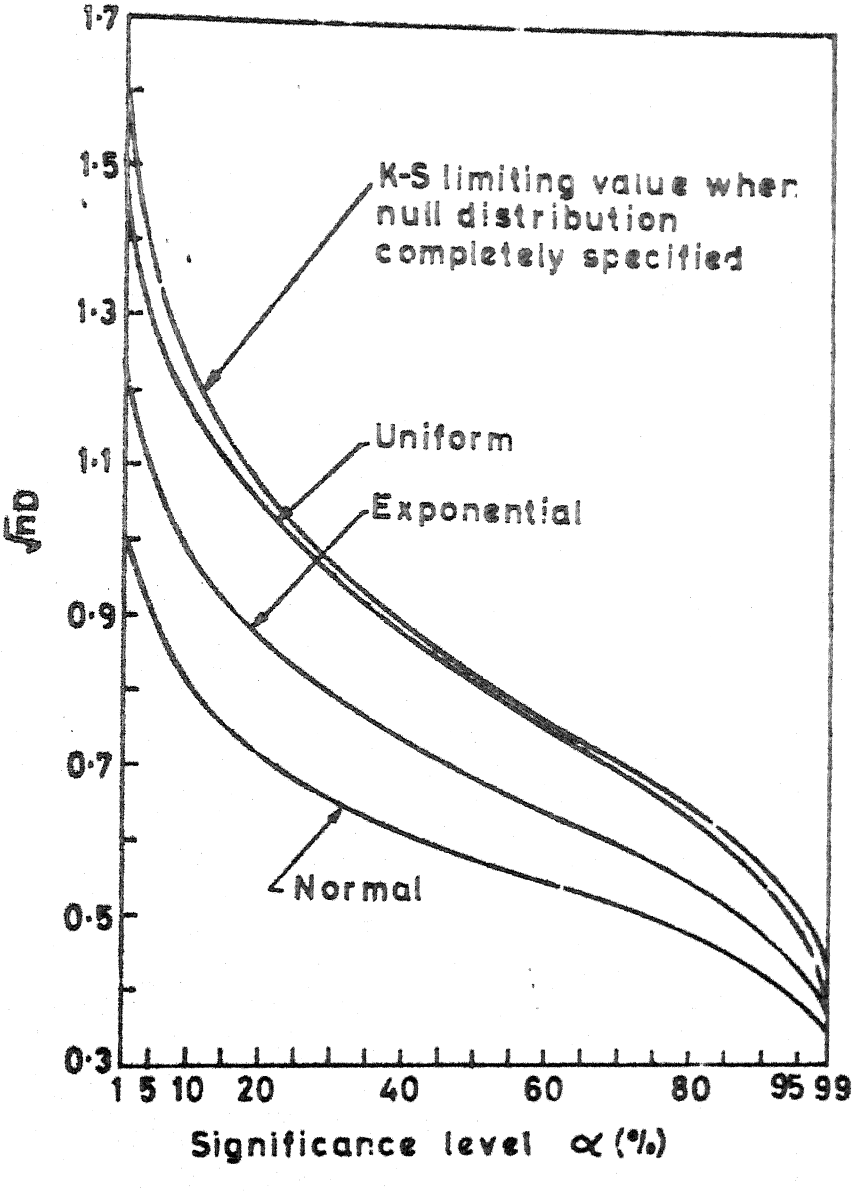


Fig.3·4 Critical values for use with K-S test when the parameters are estimated from the sample

3.5 Goodness of Fit

To test whether a given sample of n observations follows an assumed distribution, the K-S statistic D is computed and then compared with the critical values given in the tables. For example, to test whether the data of 1:3 mix follows normal distribution, parameters of the normal distribution are estimated from the data. K-S test, yields the value of statistic D = 0.0366. From Fig. 3.4 it is found that with $\sqrt{n}D = 0.4744$, normal distribution can be accepted at a maximum significance level of 82 percent. It is to be noted that the p-level found by this test is unique for a particular data and for a particular null distribution as opposed to the χ^2 goodness of fit test which changes with the number of classes. One of the major advantages of K-S test is that there is no ambiguity about the p-level of the test. To test whether the same data follows lognormal distribution, mean and standard deviation are estimated after taking natural logarithm of the data points. K-S test for normal distribution is then performed giving the value of statistic D = 0.0371. Thus lognormal distribution can be accepted at a maximum significance level of 80.5 percent. The K-S test results of the three different mix data are given in Table 3.8. Now, comparing

Table 3.8 : K-S Test Result of Three Mix Data of Mortar Strength

Mix designation	n	Null distribution					
		Normal			Lognormal		
		D	$\sqrt{n}$ D	p-level(%)	D	$\sqrt{n}$ D	p-level(%)
1:3	168	0.0366	0.4744	82.0	0.0371	0.4809	80.5
1:4	168	0.0641	0.8308	8.5	0.0468	0.6066	45.0
1:5	156	0.0839	1.0479	0.5	0.0695	0.8681	6.0

the p-levels, the distribution which gives a higher p-level can be accepted as a better fit. It can be noticed from Table 3.8 that normal distribution appears to be a better fit to the 1:3 mix data whereas lognormal distribution appears to be better in case of 1:4 and 1:5 mix data. Had the parameters been completely specified or known from past experience or from common sense, the direct comparison of D values for the two distributions could be done and the distribution which gives a lower value of D would be the better fit over the other distribution. It can be observed from Tables 3.4 and 3.8 that lognormal distribution appears to be better fit according to chi-square test whereas normal distribution appears to be better fit according to K-S test on 1:3 mix data. The decision about the acceptance of the better fitted distribution has changed in this case. Similar result is observed for 1:5 mix data. However, both chi-square and K-S test have shown that lognormal distribution is a better model to represent 1:4 mix data than normal distribution.

CHAPTER 4

RELIABILITY ANALYSIS OF REINFORCED BRICK BEAMS

4.1 Introduction

Strength of masonry (f_w) is a random variable since it depends on other variables such as brick and mortar strength, thickness of mortar joint, joint layout etc. as discussed in Chapter 2. Macchi (37) adopted normal distribution for strength of masonry. It was reported by Foster (3) that truncated normal and lognormal distributions were argued by Beech as better models to represent masonry strength. From the statistical analysis of simulated samples presented in Chapter 2, strength of masonry is taken as normally distributed random variable in computation of probability of failure. Statistical analysis of strength of high yield strength deformed bars are presented in the following section.

Reliability analysis of reinforced brick beam section at ultimate flexural strength is presented in this chapter. Moment capacity of a RBB section is a function of strengths of masonry and steel, area of tensile steel and geometric properties of the section. Strength of masonry (f_w) and strength of steel (f_y) are considered as random variables whereas rest of the parameters are treated deterministically.

A general formulation for computation of probability of failure of RBB section for deterministic and probabilistic load is presented considering under-reinforced and over-reinforced failures. A Monte Carlo simulation approach is also presented to study the probability distribution of moment capacity of RBB section for probabilistic variations of strengths of materials. Probability density function $f_X(x)$ and probability distribution function $F_X(x)$ are denoted as $f_X(X)$ and $F_X(X)$ respectively to avoid mixing up of different symbols through out the thesis, i.e., differentiation between random variable X and its state variable x is not done.

4.2 Statistical Analysis of Strength of Steel

High yield strength deformed (HYSD) bars of 8 mm diameter were selected to study the variability of ultimate strength. Samples were cut from two different lots of 8 mm dia HYSD bars and tested for ultimate strength in Universal Testing Machine. Samples of two different lots denoted as S1 and S2 can be taken as the representative of two fixed lots. Statistical analysis of data of ultimate strength f_u is given in Table 4.1. After combining the strength data of S1 and S2 (denoted as S3), statistical analysis is carried out and given in Table 4.1 . Sample S3 can be taken as the representative of mixed lot supplied by different contractors at a project site. Many builders and engineering

Table 4.1 : Statistical Analysis of Strength of HYSD Bars

Sample designation	Parameter N/mm^2	n	Mean N/mm^2	C.O.V. δ(%)	Coeff.of skewness C_s	Coeff.of kurtosis C_k	Range	
							Lower	Upper
S1 (8Φ)	f_u	37	558.71	1.68	0.889	4.997	538.00	589.78
S2 (8Φ)	f_u	34	606.28	1.33	0.595	2.285	595.15	622.73
S3 (8Φ) Combination of S1 and S2	f_u	71	581.49	4.38	0.052	1.408	538.00	622.73
Random lot	f_u	168	566.13	12.28	0.603	4.434	382.59	845.62
	f_y	168	449.15	11.42	-0.171	4.939	245.25	605.28

supervisors had sent sets of about 3 to 6 HYSD bars of diameter varying from 8 mm to 28 mm for quality analysis at I.I.T. Kanpur. The sample containing these sets can be considered completely random and is denoted as random lot. Results of statistical analysis of ultimate strength f_u, and proof strength f_y of random lot are given in Table 4.1.

The coefficient of variation of ultimate strength is found to be within 1.68 percent for two fixed lots which signifies a very good quality control in a particular production batch. The mean values of the two lots are found to be 558.71 N/mm^2 and 606.28 N/mm^2. A higher coefficient of variation of 4.38 percent is observed in the mixed lot. This again reflects a consistent quality control between different batches. Coefficient of variation observed in the random lot is 12.28 percent for the ultimate strength data and it is equal to 11.42 percent for the proof strength. Histograms of ultimate strength and proof strength of HYSD bars of random lot are shown in Fig. 4.1 and Fig. 4.2 respectively. Chi-square test is carried out to the strength data. Ultimate strength of steel followed normal distribution at 0.5 percent significance level while normal distribution can be accepted for proof strength of steel at 1 percent significance level. In both cases normal distribution appears as better representative model than lognormal distribution. The coefficient of variation of

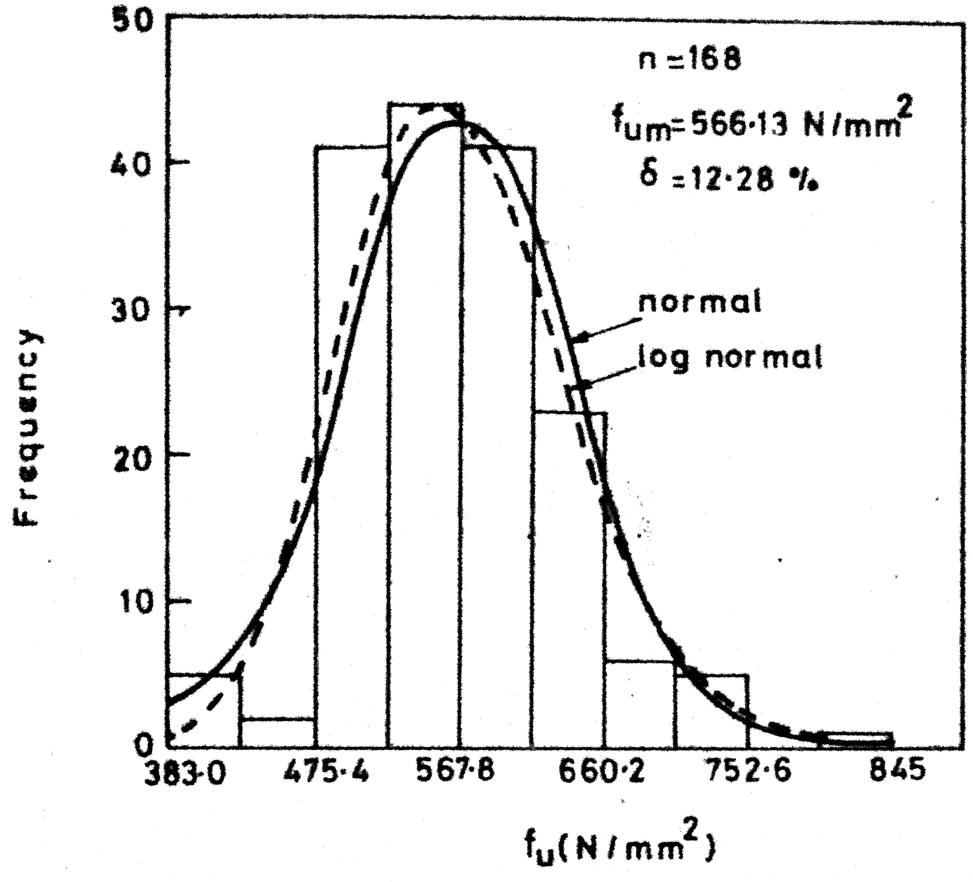


Fig. 4·1 Histogram of ultimate strength of steel

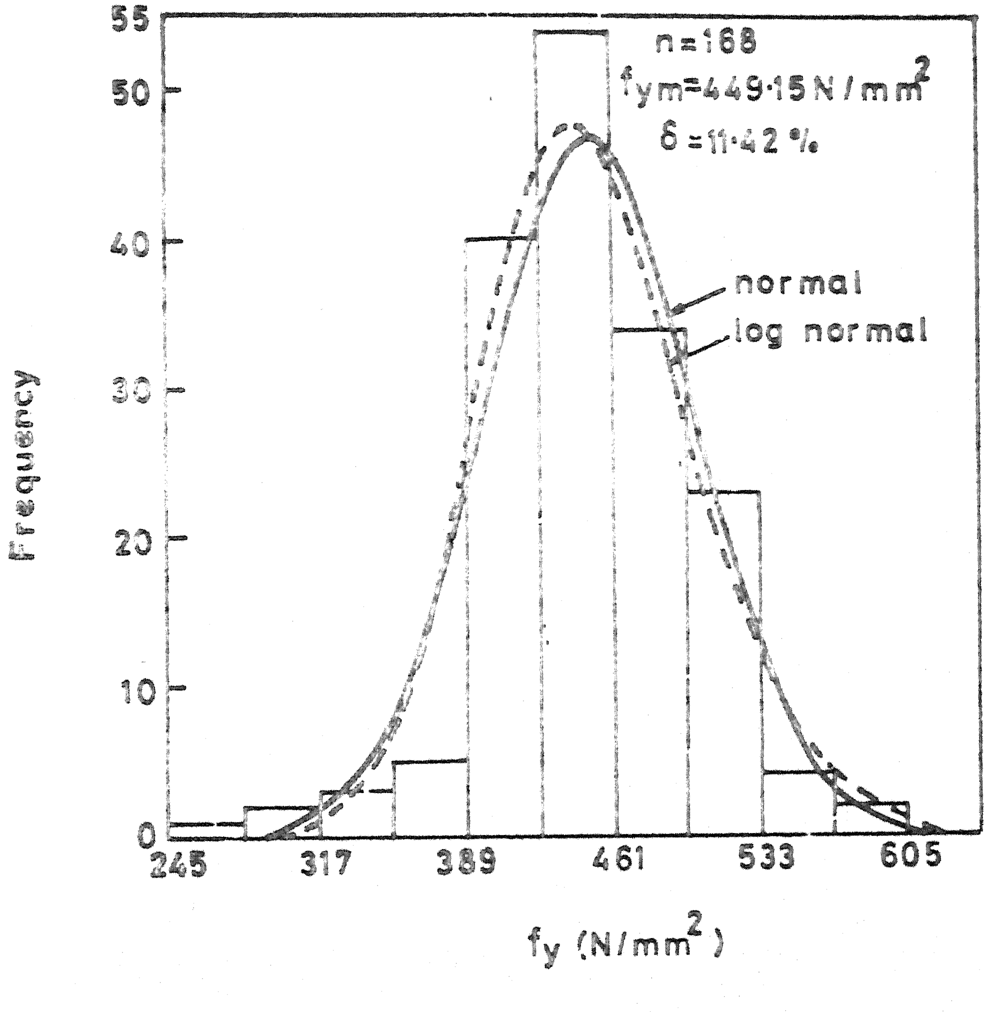


Fig.4·2 Histogram of proof strength of steel

proof strength of HYSD bars could vary between 5 to 12 percent in actual construction.

4.3 Equations for Determination of Ultimate Strength of RBB Section

The equations for calculating moment capacity are derived from the following assumptions:

(i) Plane sections remain plane after bending.

(ii) Tensile strength of masonry is ignored and tensile force is resisted by reinforcing steel alone.

(iii) A rectangular stress block is assumed. The width and depth of stress block are $k_1 f_w$ and $k_2 \bar{a}$ where

f_w = strength of masonry

$\bar{a}$ = depth of neutral axis

k_1, k_2 = constants.

(iv) Maximum strain in brickwork ϵ_{wmax} is limited to 0.003 and maximum strain in steel at failure is given by

$$\epsilon_s = 0.002 + \frac{f_y}{1.15\ E_s} \qquad (4.1)$$

where f_y = yield or proof strength of steel

E_s = Young's modulus of steel.

Details of stress block are shown in Fig. 4.3. When the failure is governed by the yielding of tensile reinforcement, the section is said to be under-reinforced. Moment capacity M_r, for an under-reinforced section is given by

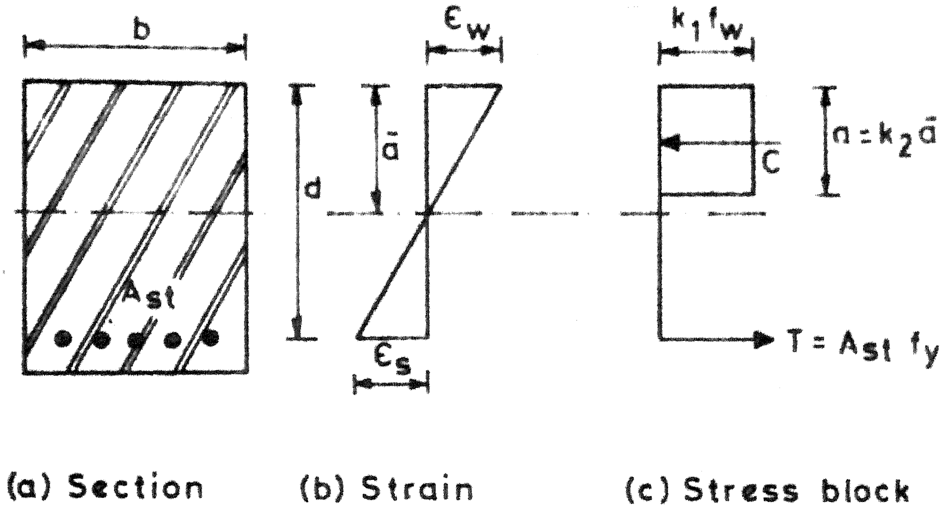


Fig. 4·3 Details of stress block

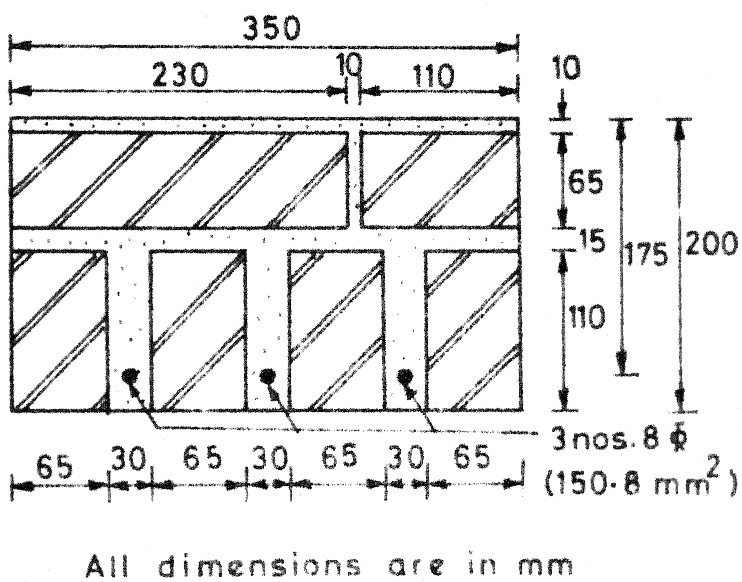


Fig. 4·4 Details of RBB section

$$M_r = A_{st}\, f_y\, jd \tag{4.2}$$

where A_{st} = area of tensile steel

f_y = yield or proof strength of steel

jd = lever arm $= (d - \frac{a}{2})$

The depth of stress block is calculated by equating total compressive force C and total tensile force T,

$$C = k_1 f_w\, ab = T = A_{st} f_y \tag{4.3}$$

which results in

$$a = \frac{A_{st}}{b} \cdot \frac{f_y}{k_1 f_w} \tag{4.4}$$

and

$$j = (1 - 0.5\, \frac{A_{st}}{bd} \cdot \frac{f_y}{k_1 f_w}) \tag{4.5}$$

The section is said to be over-reinforced when the failure is governed by crushing of masonry. The strain in masonry reaches its limiting value whereas strain in steel is less than its limiting value. At balanced failure, i.e., both the strains in masonry and steel reach their limiting values simultaneously and depth of neutral axis at balanced failure $\bar{a}_{lim}$ can be calculated as

$$\frac{\bar{a}_{lim}}{d - \bar{a}_{lim}} = \frac{0.003}{0.002 + \dfrac{f_y}{1.15\, E_s}} \tag{4.6}$$

For $E_s = 200000\ N/mm^2$, the above equation becomes

$$\bar{a}_{lim} = k_3 d \tag{4.7}$$

where

$$k_3 = \frac{690}{1150 + f_y}$$

Thus, moment capacity of an over-reinforced section is given by

$$M_r = (k_1 f_w).(k_2 \bar{a}_{lim}).b(d - \frac{k_2 \bar{a}_{lim}}{2}) \qquad (4.8)$$

Substituting $\bar{a}_{lim}$ from Eq. 4.7 to the above equation

$$M_r = k_1 k_2 k_3 (1 - \frac{k_2 k_3}{2})\ bd^2 f_w \qquad (4.9)$$

The condition that a RBB section is balanced or over-reinforced is given by

$$a \geq k_2 \bar{a}_{lim}$$

or $$\frac{A_{st}}{b} \cdot \frac{f_y}{k_1 f_w} \geq k_2 k_3 d$$

or $$\frac{A_{st}}{bd} \cdot \frac{f_y}{f_w} \geq k_1 k_2 k_3 \qquad (4.10)$$

where k_1, k_2 and k_3 depend on the assumptions regarding the stress block and strain limitations.

From Whitney's theory applied to reinforced concrete, $k_1 = 0.85$ and $k_2 = 0.85$. For k_1=0.85, k_2=0.85 and f_y=415 N/mm^2, Eqs. 4.2, 4.5, 4.7, 4.9 and 4.10 can be expressed in a compact form

$$M_r = A_{st} f_y d (1 - 0.59 \frac{A_{st}}{bd} \cdot \frac{f_y}{f_w}) \quad \text{if } \frac{A_{st}}{bd} \cdot \frac{f_y}{f_w} < 0.319$$

$$= 0.259\ bd^2 f_w \quad \text{if } \frac{A_{st}}{bd} \cdot \frac{f_y}{f_w} \geq 0.319 \qquad (4.11)$$

Dimensions and area of tensile steel of a RB section which will be used for reliability analysis are shown in Fig.4.4.

4.4 Probability of Failure of RBB Section

Moment capacity of a RBB section M_r is a function of strengths of masonry and steel, and geometric properties of the section and can be expressed as

$$M_r = g(f_w, f_y, b, d, A_{st}) \tag{4.12}$$

where

f_w = strength of masonry at 28 days

f_y = strength of steel

b = breadth of the section

d = effective depth of the section

A_{st} = area of tensile steel.

Since f_w and f_y are random variables, M_r becomes random variable and the section can fail either by yielding of tensile reinforcement or by crushing of masonry.

Let F be the 'Failure event' denoted by $R-S < 0$ or $R/S < 1$ where R and S represent generalized resistance and load respectively. Due to random variations, any of the following events can occur:

U - the section is under-reinforced

O - the section is over-reinforced.

Thus failure may occur under any of the following events:

F_U - the section fails as under-reinforced i.e., $(F \cap U)$

F_O - the section fails as over-reinforced i.e.; $(F \cap O)$

Since the events F_U and F_O are mutually exclusive (this assumption is justified because a section failing as under-reinforced cannot fail as over-reinforced at the same time), the probability of failure p_f can be expressed as

$$p_f = P(R-S<0) \text{ or } P(R/S<1) = P(F_U)+P(F_O) \qquad (4.13)$$

Defining p_{fu} and p_{fo} as follows, from Baye's Theorem

$$p_{fu} = P(F_U) = P(F \cap U) = P(F|U).\ P(U) \qquad (4.14)$$

$$p_{fo} = P(F_O) = P(F \cap O) = P(F|O).\ P(O) \qquad (4.15)$$

where $P(F|U)$ is the conditional probability of F given U and $P(F|O)$ is the conditional probability of F given O.

Thus, Eq. 4.13 can be written as

$$\begin{aligned} p_f &= p_{fu} + p_{fo} \\ &= P\ (F \cap U) + P(F \cap O) \\ &= P(F|U).P(U) + P(F|O).P(O) \qquad (4.16) \end{aligned}$$

Probability of failure p_f can be computed as the sum of (i) the probability of failure of the section as an under-reinforced (p_{fu}) and (ii) the probability of failure of the section as an over-reinforced (p_{fo}).

4.5 Reliability Analysis Formulation for Probalistic Variations of Strengths of Materials

4.5.1 General

The ultimate moment capacity M_r of a RBB section may be expressed in a general form as

$$M_r = K_1 A_{st} f_y d\left(1 - K_2 \frac{A_{st}}{bd} \cdot \frac{f_y}{f_w}\right)$$

$$\text{if } \frac{A_{st}}{bd} \cdot \frac{f_y}{f_w} < K_o \qquad (4.17)$$

and $$M_r = K_3\, bd^2 f_w \qquad \text{if } \frac{A_{st}}{bd} \cdot \frac{f_y}{f_w} \geq K_o \qquad (4.18)$$

where f_w, f_y, b, d and A_{st} are defined as in Eq. 4.12. K_1, K_2, K_3 and K_o are positive constants which depend on the assumptions regarding the stress block and strain limitations.

The strengths of masonry and steel are treated as random variables whereas rest of the parameters are treated as deterministic constants in the following derivation. From Eqs. 4.17 and 4.18, the events are identified as follows:

Event U : Under-reinforced Case

The condition, that a section is under-reinforced is given by

$$\frac{A_{st}}{bd} \cdot \frac{f_y}{f_w} < K_o \qquad (4.19)$$

and the corresponding moment capacity can be computed by Eq. 4.17.

Event O : Over-reinforced case

The condition that a section is over-reinforced is given by

$$\frac{A_{st}}{bd} \cdot \frac{f_y}{f_w} \geq K_o \tag{4.20}$$

and the corresponding moment capacity can be computed by Eq. 4.18.

In both the cases, it can be noticed that the resistance M_r is a function of random variables f_w and f_y, and thus becomes a random variable.

4.5.2 Computation of p_f for deterministic external moment

Probability of failure p_f, for deterministic external moment M_e is given by

$$p_f = P(M_r \leq M_e) \tag{4.21}$$

For convenience, Eqs. 4.17 and 4.18 is written in the following form

$$M_r = c_1 f_y \left(1 - c_2 \frac{f_y}{f_w}\right) \text{ if } \frac{f_y}{f_w} < c_o \tag{4.22}$$

$$= c_3 f_w \qquad \text{if } \frac{f_y}{f_w} \geq c_o \tag{4.23}$$

where

$$c_o = K_o \frac{bd}{A_{st}} \tag{4.24}$$

$$c_1 = K_1 \, A_{st} \, d \tag{4.25}$$

$$c_2 = K_2 \frac{A_{st}}{bd} \tag{4.26}$$

$$c_3 = K_3 \, bd^2 \tag{4.27}$$

Following assumptions are made for the derivation of p_f :

(i) f_w and f_y are independent positive random variables ranging from zero to infinity,

(ii) c_o, c_1, c_2 and c_3 are all positive constants,

(iii) value of c_o is such that M_r is always positive quantity.

All the above assumptions are justified since strength of masonry and steel are positive random variables and all the constants c_o, c_1, c_2 and c_3 are functions of positive valued parameters. Value of c_o is determined from K_o which guarantees that M_r will always be positive.

From Eqs. 4.22 and 4.23, p_f given by Eq. 4.21 is expressed as

$$\begin{aligned} p_f &= P\ (M_r \leq M_e) \\ &= P(\{ c_1 f_y (1 - c_2 \frac{f_y}{f_w}) \leq M_e \} \cap \{ \frac{f_y}{f_w} < c_o \}) \\ &\quad + P(\{ c_3 f_w \leq M_e \} \cap \{ \frac{f_y}{f_w} \geq c_o \}) \\ &= p_{fu} + p_{fo} \end{aligned} \tag{4.28}$$

It should be noted that the first part of the above equation is p_{fu} and the second term is p_{fo} as can be seen from Eq.4.16. The event $\{c_1 f_y(1 - c_2\frac{f_y}{f_w}) \leq M_e\}$ of Eq. 4.28 can be written as

$$f_w\ (c_1 f_y - M_e) \leq c_1 c_2 f_y^2 \tag{4.29}$$

The right hand part $c_1 c_2 f_y^2$ of the above inequality is a positive quantity. Since $(c_1 f_y - M_e)$ can be negative or positive depending on f_y and M_e, the inequality given by Eq. 4.29 resolves into two conditional inequalities. The condition $(c_1 f_y - M_e) > 0$ implies $f_w \leq \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e}$ and $(c_1 f_y - M_e) < 0$ implies $f_w > 0$ which is trivially satisfied. Thus,

$$f_w \leq \frac{c_1 c_2\, f_y^2}{c_1 f_y - M_e} \qquad \text{if} \quad f_y > \frac{M_e}{c_1}$$

$$f_w > 0 \qquad \text{if} \quad f_y < \frac{M_e}{c_1}$$

Therefore, first part of Eq. 4.28 becomes

$$\begin{aligned} p_{fu} = {} & P\left(\{f_w \leq \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e}\} \cap \{ f_y < c_o f_w\} \;\middle|\; \{ f_y > \frac{M_e}{c_1}\} \right) \\ & \cdot P(f_y > \frac{M_e}{c_1}) \\ & + P(\{ f_w > 0\} \cap \{f_y < c_o f_w\} \mid \{f_y < \frac{M_e}{c_1}\}) \cdot P(f_y < \frac{M_e}{c_1}) \\ = {} & P(\{ f_w \leq \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e}\} \cap \{f_w > \frac{f_y}{c_o}\} \cap \{f_y > \frac{M_e}{c_1}\}) \\ & + P(\{f_w > 0\} \cap \{f_w > \frac{f_y}{c_o}\} \cap \{f_y < \frac{M_e}{c_1}\}) \end{aligned} \tag{4.30}$$

For $\dfrac{c_1 c_2 f_y^2}{c_1 f_y - M_e} < \dfrac{f_y}{c_o}$, the event

$$\{f_w \leq \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e}\} \cap \{f_w > \frac{f_y}{c_o}\}$$

vanishes. Thus the above event can be written as

$$\{ \frac{f_y}{c_o} < f_w \leq \max \left(\frac{f_y}{c_o} , \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e} \right) \}$$

Therefore, Eq. 4.30 becomes

$$p_{fu} = P \left(\{ \frac{f_y}{c_o} < f_w \leq \max \left(\frac{f_y}{c_o} , \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e} \right) \} \cap \{ \frac{M_e}{c_1} < f_y < \infty \} \right)$$

$$+ P (\{ \frac{f_y}{c_o} < f_w < \infty \} \cap \{ 0 < f_y < \frac{M_e}{c_1} \}) \qquad (4.31)$$

The region of integration in f_w **and f_y plane** is shown in Fig. 4.5. Eq. 4.31 can be expressed in integral form as

$$p_{fu} = \int_{\frac{M_e}{c_1}}^{\infty} \left[\int_{\frac{f_y}{c_o}}^{\max\left(\frac{f_y}{c_o} , \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e}\right)} f_{fw}(f_w)\, df_w \right] f_{fy}(f_y)\, df_y$$

$$+ \int_{0}^{\frac{M_e}{c_1}} \left[\int_{\frac{f_y}{c_o}}^{\infty} f_{fw}(f_w)\, df_w \right] f_{fy}(f_y)\, df_y \qquad (4.32)$$

where $f_{fw}(f_w)$ and $f_{fy}(f_y)$ are the probability density functions of independent random variables f_w and f_y respectively.

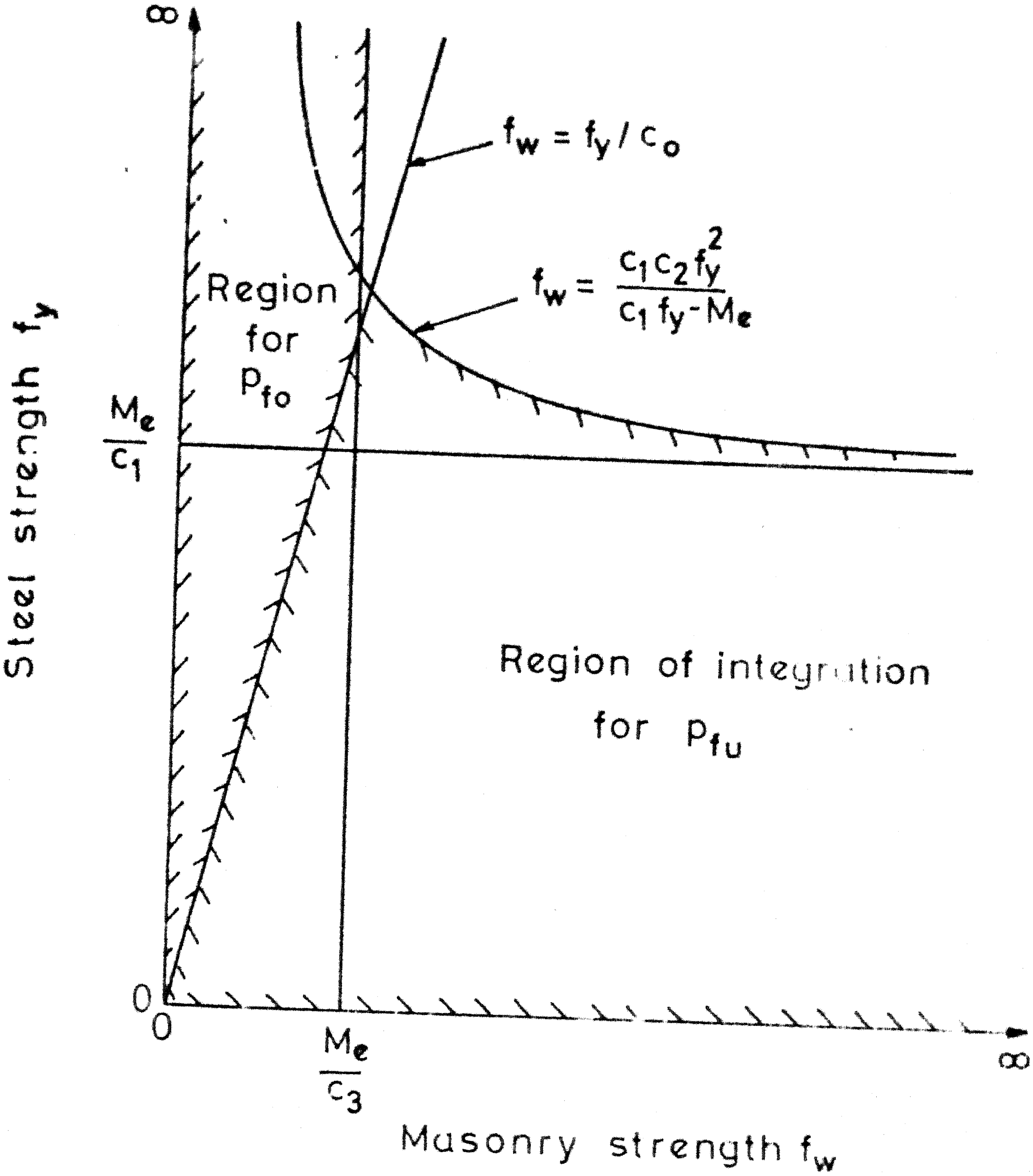


Fig.4·5 Region of integration on f_w and f_y plane

Second part of Eq. 4.28 can be written as

$$p_{fo} = P\left(\{ f_w \leq \frac{M_e}{c_3} \} \cap \{ f_w \leq \frac{f_y}{c_o} \} \right)$$

$$= P\left(0 < f_w \leq \min\left(\frac{f_y}{c_o}, \frac{M_e}{c_3}\right) \right) \qquad (4.33)$$

The region of integration is shown in Fig. 4.5. Eq. 4.33 can be expressed in integral form as

$$p_{fo} = \int_0^{\infty} \left[\int^{\min\left(\frac{f_y}{c_o}, \frac{M_e}{c_3}\right)} f_{fw}(f_w)\, df_w \right] f_{fy}(f_y)\, df_y \qquad (4.34)$$

where $f_{fw}(f_w)$ and $f_{fy}(f_y)$ are the probability density functions of independent random variables f_w and f_y respectively.

It should be noted that the expressions derived for p_f given by Eq. 4.28 represents the probability distribution function $F_{Mr}(M_e)$ of random variable M_r and thus can be expressed as

$$F_{Mr}(M_e) = P(M_r \leq M_e)$$

$$= p_{fu} + p_{fo} \qquad (4.35)$$

where p_{fu} and p_{fo} has to be calculated from Eq. 4.32 and Eq. 4.34.

4.5.3 Computation of p_f for probabilistic external moment

Moment capacity M_r is a random variable because the strengths of masonry and steel are random variables.

Loads coming on a structure is also a random variable. If load is taken as probabilistic, the external moment M_e becomes random variable. It will be assumed that load and resistance are independent, i.e., M_e and M_r are independent random variables. The probability of failure of a RBB section for probabilistic external moment M_e can be expressed as

$$p_f = P(M_r \leq M_e)$$

$$= \int_D \int f_{M_r}(M_r)\, f_{M_e}(M_e)\, dM_r\, dM_e \qquad (4.36)$$

where M_r and M_e are independent random variables with probability density functions $f_{M_r}(M_r)$ and $f_{M_e}(M_e)$ respectively. Assuming external moment M_e as positive random variable ranging from 0 to ∞, Eq. 4.36 becomes

$$p_f = \int_0^\infty \left[\int_0^{M_e} f_{M_r}(M_r)\, dM_r \right] f_{M_e}(M_e)\, dM_e$$

$$= \int_0^\infty F_{M_r}(M_e)\, f_{Me}(M_e)\, dM_e \qquad (4.37)$$

where $F_{M_r}(M_e)$ has to be calculated by Eq 4.35.

4.6 Computation of p_f for Deterministic External Moment When Strengths of Materials Follow Normal Distribution

Expressions for probability of failure given by Eq. 4.32 and Eq.4.34 were derived under the assumption that f_w and f_y are independent positive random variables ranging from zero to infinity. Although strength of masonry f_w

and strength of steel f_y cannot be negative, normal distribution was found to be suitable model for f_w and f_y as discussed earlier. The probability of a random variable X, following normal distribution with coefficient of variation δ, to be negative is given by

$$P(X < 0) = \int_{-\infty}^{0} \frac{1}{s\sqrt{(2\pi)}} \exp\left[-\frac{1}{2}\left(\frac{x-X_m}{s}\right)^2\right] dx$$

$$= \phi\left(-1/\delta\right) \tag{4.38}$$

For δ as high as 0.25, the probability of the random variable being negative is 3.71×10^{-5} which is very small for practical purposes. Thus, normal probability density functions can be used in Eq. 4.32 and Eq. 4.34 for computation of p_f when the coefficient of variation is limited to 25 percent without introducing much error.

Substituting normal density functions for f_w and f_y, Eq. 4.32 becomes

$$p_{fu} = \int_{\frac{M_e}{c_1}}^{\infty} \left[\int_{\frac{f_y}{c_o}}^{\max\left(\frac{f_y}{c_o},\ \frac{c_1 c_2 f_y^2}{c_1 f_y - M_e}\right)} \frac{1}{\sqrt{2\pi}\, s_w} \exp\left(-\frac{1}{2}\left(\frac{f_w-f_{wm}}{s_w}\right)^2\right) df_w \right] \cdot \frac{1}{\sqrt{2\pi}\, s_y} \exp\left(-\frac{1}{2}\left(\frac{f_y-f_{ym}}{s_y}\right)^2\right) df_y$$

$$+ \int_{0}^{\frac{M_e}{c_1}} \left[\int_{\frac{f_y}{c_o}}^{\infty} \frac{1}{\sqrt{2\pi}\, s_w} \exp\left(-\frac{1}{2}\left(\frac{f_w-f_{wm}}{s_w}\right)^2\right) df_w \right] \cdot \frac{1}{\sqrt{2\pi}\, s_y} \cdot \exp\left(-\frac{1}{2}\left(\frac{f_y-f_{ym}}{s_y}\right)^2\right) df_y \tag{4.39}$$

where

f_{wm}, s_w = mean and standard deviation of f_w respectively

f_{ym}, s_y = mean and standard deviation of f_y respectively.

Eq. 4.39 can be expressed in a simplified form as

$$p_{fu} = \int_{\frac{M_e}{c_1}}^{\infty} [\phi(\max(z_1, z_2)) - \phi(z_1)] \cdot \frac{1}{\sqrt{2\pi}\, s_y} \exp\left(-\frac{1}{2}\left(\frac{f_y - f_{ym}}{s_y}\right)^2\right) df_y$$

$$+ \int_{0}^{\frac{M_e}{c_1}} [1 - \phi(z_1)] \cdot \frac{1}{\sqrt{2\pi}\, s_y} \exp\left(-\frac{1}{2}\left(\frac{f_y - f_{ym}}{s_y}\right)^2\right) df_y \quad (4.40)$$

where

$$z_1 = \frac{f_y/c_o - f_{wm}}{s_w}$$

$$z_2 = \left(\frac{c_1 c_2 f_y^2}{c_1 f_y - M_e} - f_{wm}\right) / s_w$$

$\phi(.)$ = standardized normal distribution function.

Similarly, substituting normal density functions for f_w and f_y, Eq. 4.34 becomes

$$p_{fo} = \int_0^{\infty} \left[\int_0^{\min\left(\frac{f_y}{c_o}, \frac{M_e}{c_3}\right)} \frac{1}{\sqrt{2\pi}\, s_w} \exp\left(-\frac{1}{2}\left(\frac{f_w - f_{wm}}{s_w}\right)^2\right) df_w \right]$$

$$\cdot \frac{1}{\sqrt{2\pi}\, s_y} \exp\left(-\frac{1}{2}\left(\frac{f_y - f_{ym}}{s_y}\right)^2\right) df_y \quad (4.41)$$

where f_{wm}, s_w, f_{ym} and s_y are defined in Eq. 4.39.

The above equation can be expressed in a simplified form as

$$p_{fo} = \int_0^{\infty} [\, \phi\,(\min(z_1, z_3)) - \phi\,(-\, f_{wm}/s_w)\,] \cdot \frac{1}{\sqrt{2\pi}\, s_y} \exp\left(-\frac{1}{2}\left(\frac{f_y - f_{ym}}{s_y}\right)^2\right) df_y \quad (4.42)$$

where $z_3 = (M_e/c_3 - f_{wm})/s_w$

and other variables are as defined in Eq. 4.40.

Probability of failure p_f thus can be obtained by summing the probabilistics p_{fu} and p_{fo}. Probability distribution function $F_{Mr}(M_e)$ is given by Eq. 4.35 where p_{fu} and p_{fo} are to be calculated from Eqs. 4.40 and 4.42 respectively when both f_w and f_y follow normal distribution.

RBB section shown in Fig. 4.4 is selected to study the variation of probability of failure with coefficients of variation of strength of masonry (δ_{fw}) and steel (δ_{fy}). Following are the details of the RBB section:

b = 350 mm , d = 175 mm , A_{st} = 150.8 mm^2 (3 # 8 Φ).

Strengths of masonry and steel are taken as

f_w	Normal	f_{wm} = 8.96 N/mm^2
f_y	Normal	f_{ym} = 449.15 N/mm^2

Using the mean values of f_w and f_y, the mean moment capacity M_{rm} is computed from Eq. 4.11,

M_{rm} = 10.989 kNm .

For $\delta_{fy} = 0.05$, probability of failure versus δ_{fw} for different M_{rm}/M_e is shown in Fig. 4.6(a). Probability of failure p_f increases with increase in δ_{fw} and decreases with increase in M_{rm}/M_e ratio. When δ_{fy} is increased to 0.1142, p_f increases for same f_w and M_{rm}/M_e and is shown in Fig. 4.6(b). The increase in p_f is found to be dominant upto δ_{fy}=0.14, and after that increase in p_f is found to be marginal.

Effect of load factor M_{rm}/M_e on p_{fu}, p_{fo} and p_f for different combinations of δ_{fy} and δ_{fw} is shown in Figs.4.7 and 4.8. It can be noticed that in some region p_f is dominated by under-reinforced probability p_{fu} whereas in other region p_f is dominated by over-reinforced probability p_{fo}. In all the cases, a region is observed where p_f is dominated by both p_{fu} and p_{fo}. This region varies for different combinations of δ_{fy} and δ_{fw} as shown in Figs. 4.7 and 4.8. In this region, p_{fu} and p_{fo} are of the same order whereas in other region p_f is governed by either p_{fu} or p_{fo} depending on their order of magnitudes. It is found that p_{fu} and p_{fo} are sensitive to the relative magnitude of δ_{fy}, δ_{fw} and M_{rm}/M_e ratio, other variables remaining same.

The balanced area of steel A_{stb}, for the section shown in Fig. 4.4 is calculated from Eq. 4.11 using the mean values of f_w and f_y and is given by

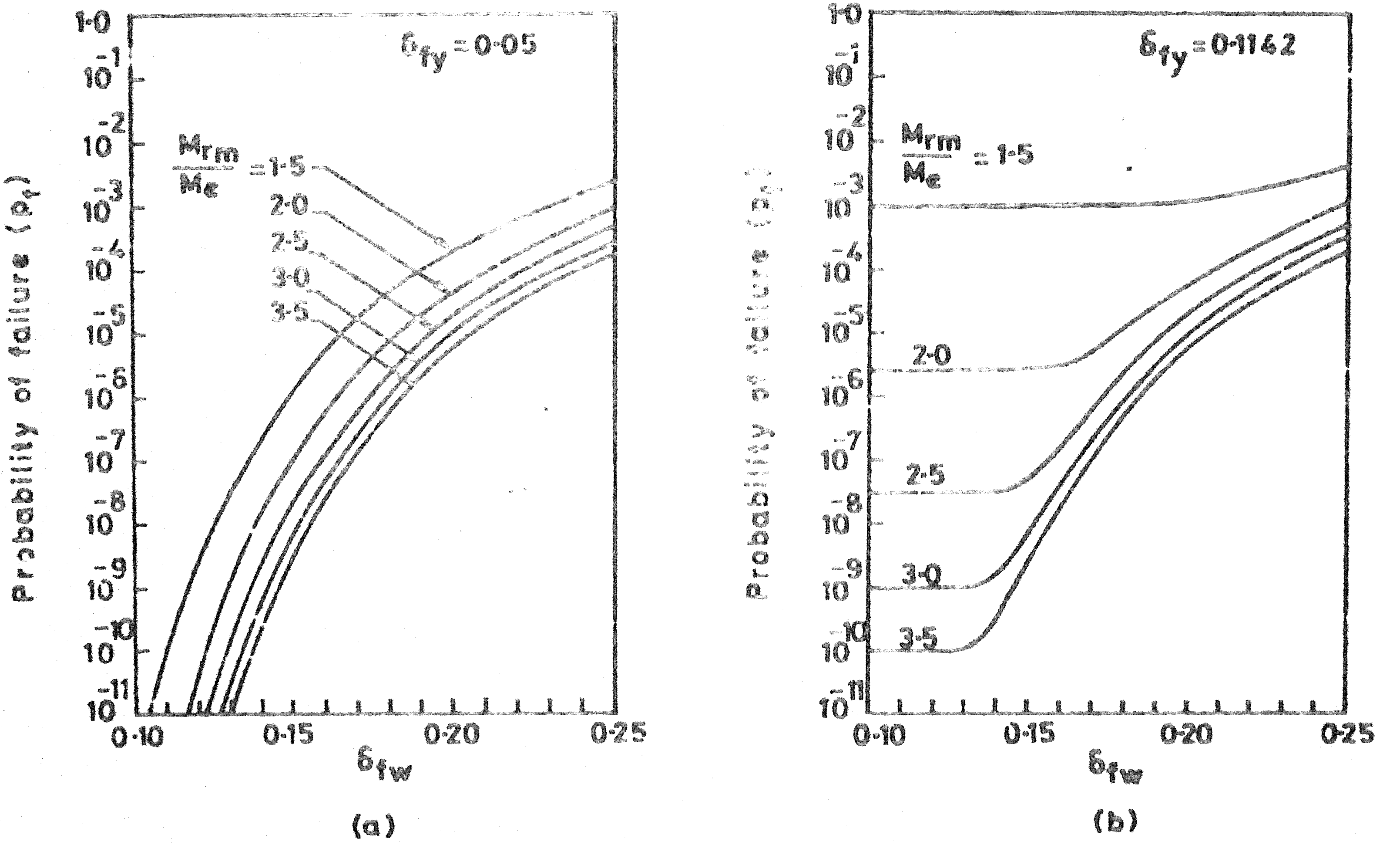


Fig. 4·6 Effect of δ_{fw} on probability of failure

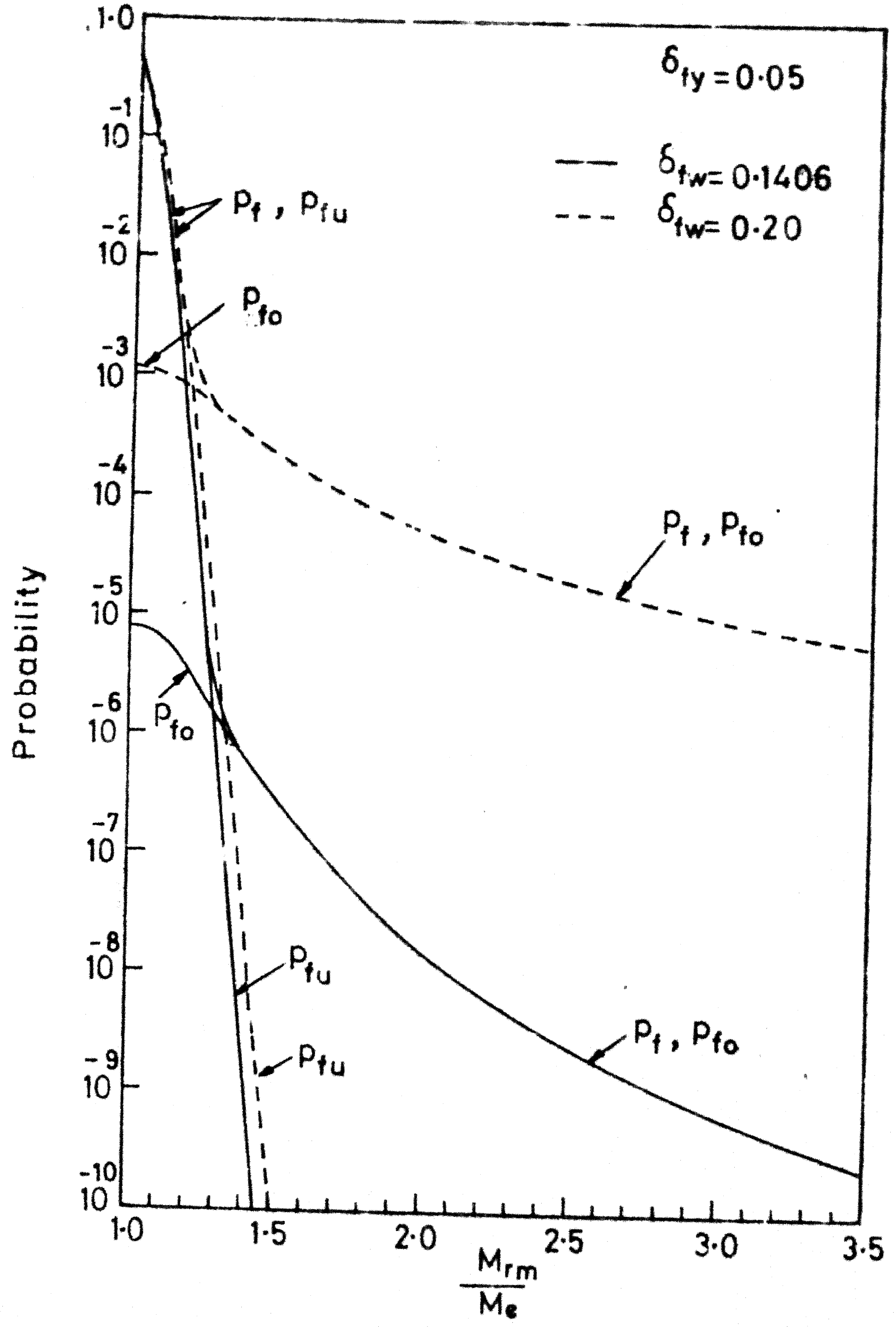


Fig. 4·7 Effect of load factor on probability of failure

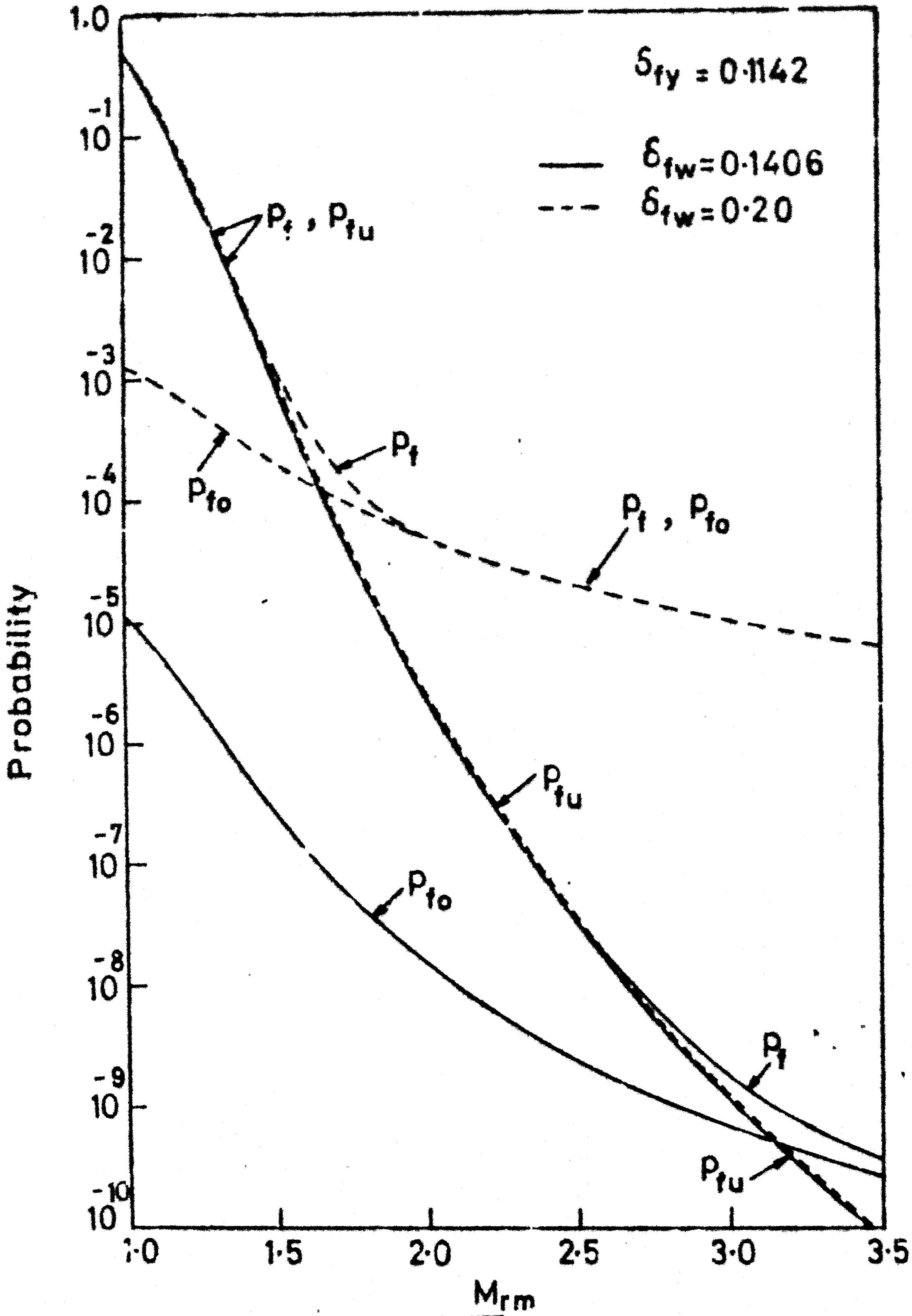
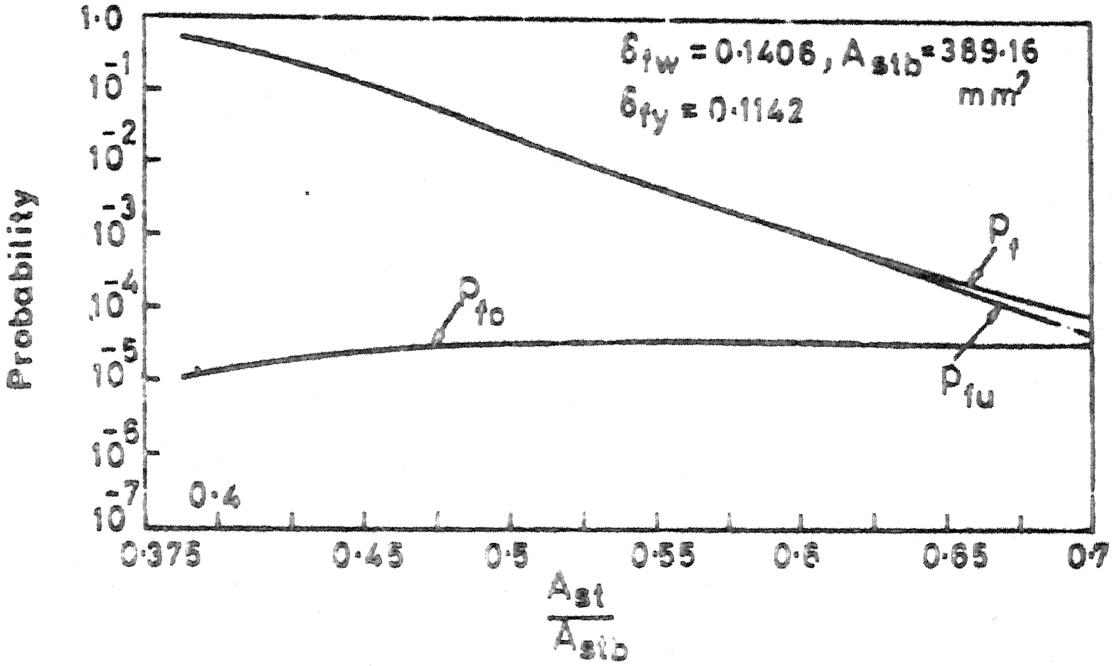


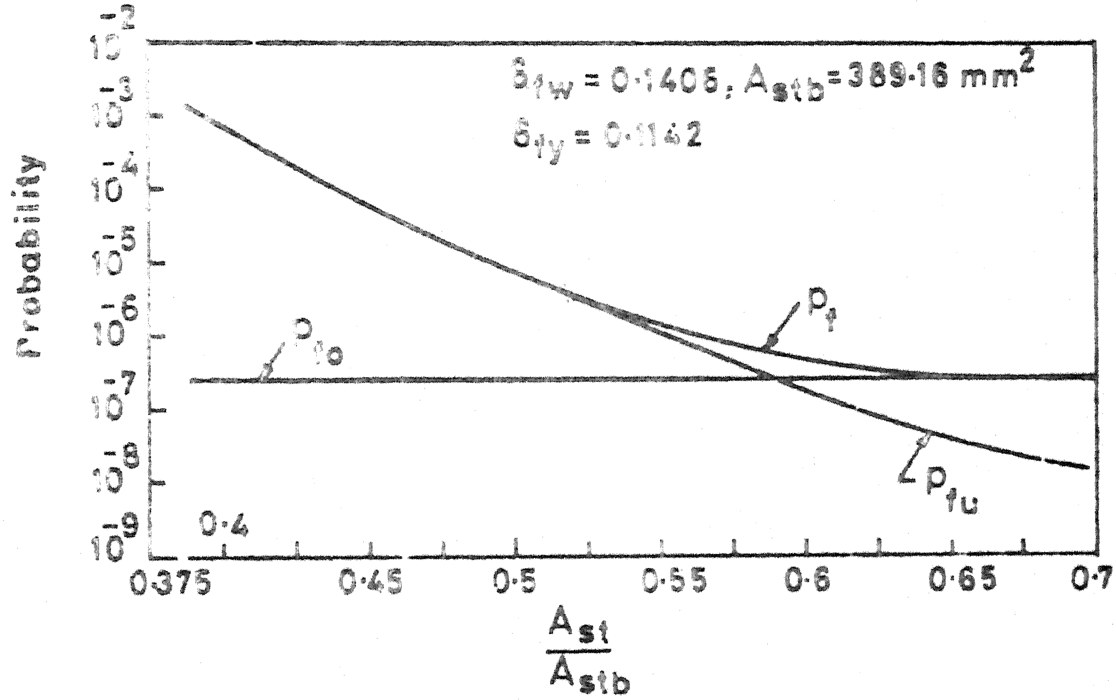
Fig.4·8 Effect of load factor on probability of failure

$$A_{stb} = 389.16 \text{ mm}^2 \text{ and } \frac{A_{st}}{A_{stb}} = 0.387.$$

Mean moment capacity M_{rm} is equal to 10.989 kNm for A_{st} = 150.8 mm^2. Effect of increase in steel area on p_{fu}, p_{fo} and p_f for external moment M_e = 10.989 kNm is shown in Fig. 4.9(a). If the area of tensile steel is increased, the probability of under-reinforced failure p_{fu} decreases and probability of over-reinforced failure p_{fo} increases but the probability of failure of the section p_f decreases for the same external moment. Fig. 4.9(b) shows the effect of increase in steel area for M_e = 7.327 kNm (M_{rm}/M_e = 1.5). Thus, if area of steel actually provided is more than that required to achieve a particular reliability of the section, the design will be on safer side but the probability of brittle failure will increase. It can be noticed from Figs. 4.9(a) and 4.9(b) that if area of steel is increased to a very large value, the probability of failure becomes constant and is equal to the over-reinforced probability p_{fo} in the limiting case. The region, where the probability of over-reinforced failure will govern the overall probability of failure p_f, depends on the relative magnitude of δ_{fy}, δ_{fw}, A_{st}/A_{stb} ratio and M_{rm}/M_e ratio.



(a) For $M_{rm} / M_e = 1{\cdot}0$



(b) For $M_{rm} / M_e = 1{\cdot}5$

Fig.4·9 Effect of increase of steel area on p_f for same external moment

4.7 Computation of p_f for Probabilistic External Moment (Lognormal) when Strengths of Materials Follow Normal Distribution

Statistical analysis of typical office building floor loads was carried out by Ranganathan and Dayaratnam(110). The frequency distribution of floor load was found to follow lognormal distribution at 5 percent significance level. External moment M_e acting on floor slabs or beams thus becomes lognormally distributed random variable. Assuming lognormal distribution for M_e, Eq. 4.37 becomes

$$p_f = \int_0^{\infty} F_{Mr}(M_e) \cdot \frac{1}{\sqrt{2\pi}\,\sigma_{ln}\, M_e} \exp\left[-\frac{1}{2}\left(\frac{\ln(M_e/M_{ln})}{\sigma_{ln}}\right)^2\right] dM_e \tag{4.43}$$

where

M_{ln} and σ_{ln} = parameters of lognormal distribution

$F_{Mr}(M_e)$ = probability distribution function of M_r

$= p_{fu} + p_{fo}$

where p_{fu} and p_{fo} are given by Eqs. 4.40 and 4.42 respectively. Parameters of the lognormal distribution can be calculated as

$$\sigma_{ln} = \sqrt{(\ln(\delta_{Me}^2+1))} \tag{4.44}$$

$$M_{ln} = M_{em} \cdot \exp(-0.5\ \sigma_{ln}^2) \tag{4.45}$$

where M_{em} and δ_{M_e} are the mean and coefficient of variation of external moment M_e.

Substituting

$$w = \frac{1}{\sqrt{2}} \cdot \frac{\ln(M_e/M_{ln})}{\sigma_{ln}}$$

Eq. 4.43 can be written in a simplified form as

$$p_f = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \cdot e^{-w^2} \cdot F_{Mr}\,(M_{ln} \cdot e^{\sqrt{2}\,w\,\sigma_{ln}})\,dw \qquad (4.46)$$

where σ_{ln} and M_{ln} are given **by** Eqs. 4.44 and 4.45 respectively. The integral given above is in standard form and is evaluated using Gauss- Hermite quadrature formula (111).

Probability of failure p_f for probabistic variation of external moment is given in Table 4.2. External moment M_e is taken as lognormally distributed with mean M_{em} and coefficient of variation δ_{Me}. Mean value of M_r is calculated by Eq. 4.11 using mean values of f_w and f_y. Table 4.2 shows the variation of p_f for different δ_{Me} and M_{rm}/M_{em} ratio. Last column of Table 4.2 gives probality of failure for deterministic external moment, i.e., $\delta_{Me}=0$. Failure probability increases as the variability of external moment increases and decreases as M_{rm}/M_{em} increases. It should be noticed that the ratio M_{rm}/M_{cm} is the central safety factor and also called load factor applied to the mean values in a design problem.

Table 4.2 : Probability of Failure for Probabilistic External Moment

$\frac{M_{rm}}{M_{em}}$	Probability of failure p_f					
	Lognormal M_e with mean M_{em} and C.O.V. δ_{M_e}					Deterministic M_e
	$\delta_{M_e}=0.20$	$\delta_{M_e}=0.25$	$\delta_{M_e}=0.30$	$\delta_{M_e}=0.35$	$\delta_{M_e}=0.40$	$\delta_{M_e}=0.0$
1.5	3.00(-2)*	5.36(-2)	6.50(-2)	8.68(-2)	1.10(-1)	1.09(-3)
2.0	1.10(-3)	3.79(-3)	1.01(-2)	1.51(-2)	2.64(-2)	2.76(-6)
2.5	3.62(-5)	2.83(-4)	1.27(-3)	3.00(-3)	7.13(-3)	3.33(-8)
3.0	1.40(-6)	2.18(-5)	1.59(-4)	6.73(-4)	1.49(-3)	1.75(-9)
3.5	7.12(-8)	1.93(-6)	2.30(-5)	1.28(-4)	4.96(-4)	3.42(-10)

* 3.00(-2) should be read as 3.00×10^{-2}.

Note: $b = 350$ mm, $d = 175$ mm, $A_{st} = 150.8$ mm^2, $f_{wm}=8.96$ N/mm^2, $f_{ym}=449.15$ N/mm^2

$\delta_{fw}=0.1406$, $\delta_{fy} = 0.1142$ and $M_{rm} = 10.989$ kNm .

4.8 Monte Carlo Simulation

Monte Carlo simulation technique (78) is used to generate random samples of moment capacity M_r to study the probability behaviour and to estimate different parameters of M_r. Details of the simulation procedure is given in Chapter 2. Choice of sample size in simulation plays an important role. Generated samples of M_r is used to estimate the mean and standard deviation. Larger the sample size used, estimates of mean, standard deviation etc. will be closer to their population values. The minimum sample size required to estimate a particular parameter depends on the desired accuracy of the estimate. The minimum sample size n, for the estimate of population mean with a confidence level $(1-\alpha)$ percent, is given by (112)

$$n = k_{\alpha/2}^{2} \left(\frac{s_x}{t} \right)^{2} \tag{4.47}$$

where s_x = sample standard deviation with n-1 as denominator

t = accepted error in the estimate of mean

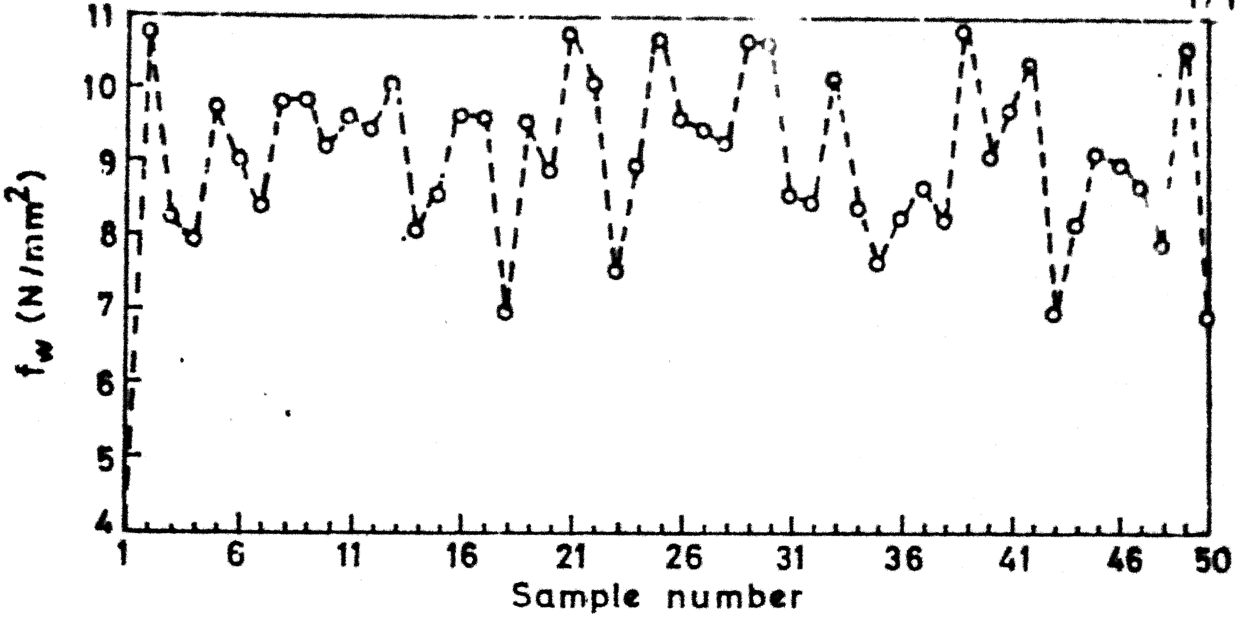
$k_{\alpha/2} = \phi^{-1}(1 - \alpha/2)$.

For large samples, s_x (with denominator n-1) gives a good estimate of the population standard deviation. For an acceptable error of $\pm$ 5 percent of the standard deviation and confidence level of 95 %, the minimum sample size required is n = 1537 and for confidence level of 99 %,

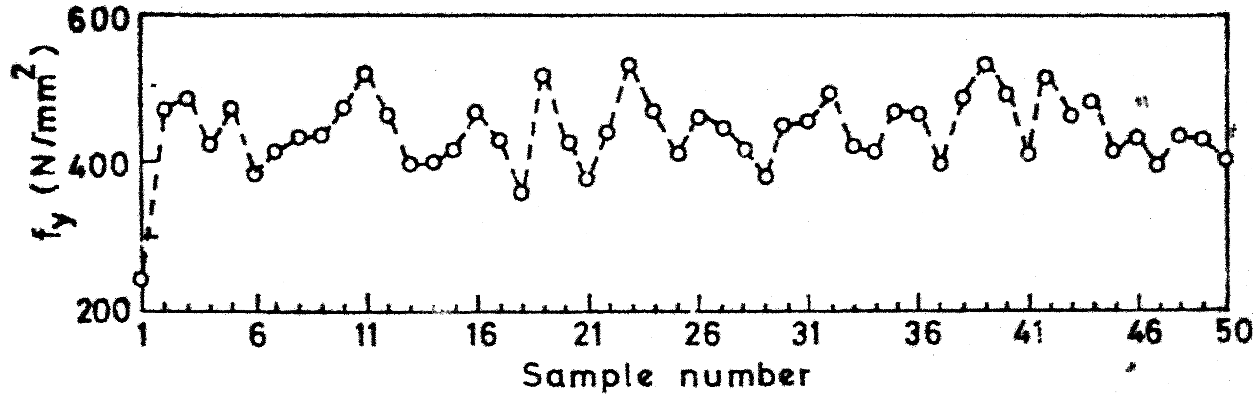
$n = 2663$. In the present work, 30000 samples are generated to study the probability behaviour of M_r.

Samples of masonry strength f_w and steel strength f_y are generated independently from their parent distributions (normal in both cases). First fifty generated samples of f_w and f_y following N (8.96, 1.26) N/mm^2 and N(449.15, 51.29)N/mm^2 are shown in Figs. 4.10(a) and 4.10(b) respectively. From each of the generated samples of f_w and f_y, sample of M_r is generated by Eq. 4.11 for the RBB section shown in Fig. 4.4. First fifty samples of M_r using the generated samples of f_w and f_y are shown in Fig.4.10(c). Since the section is highly under-reinforced deterministically ($A_{st}/A_{stb}=0.387$), the samples of M_r are dominated by the variation of steel strength f_y and not by the masonry strength f_w as seen in Fig.4.10(a), (b) and (c). As the steel area is increased towards balanced steel area, dominance of both f_w and f_y on M_r is observed. The coefficient of variation of M_r is found to be dependent on the relative magnitudes of coefficients of variation of f_w and f_y.

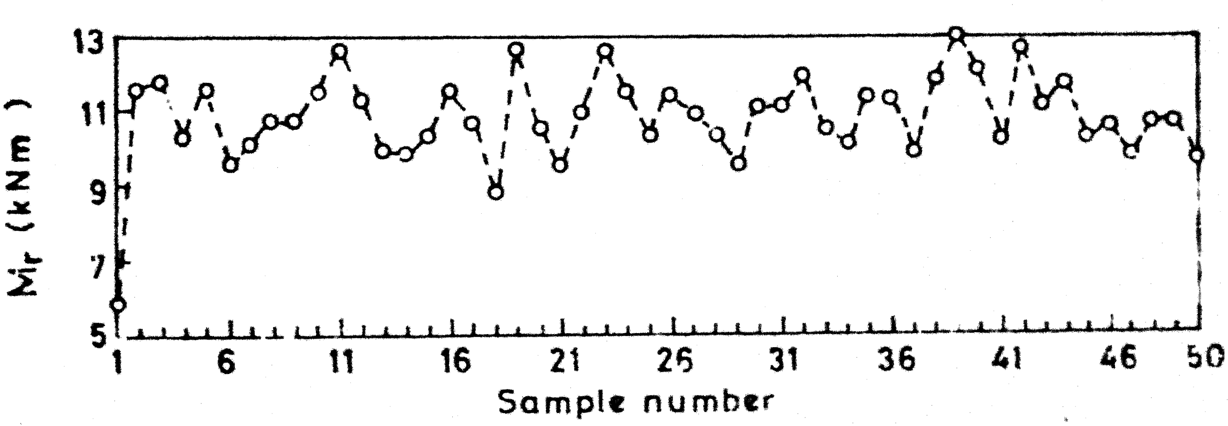
Two typical histograms of the simulated samples of M_r of the section given in Fig. 4.4 are shown in



(a) Generated samples of f_w N (8·96, 1·26)



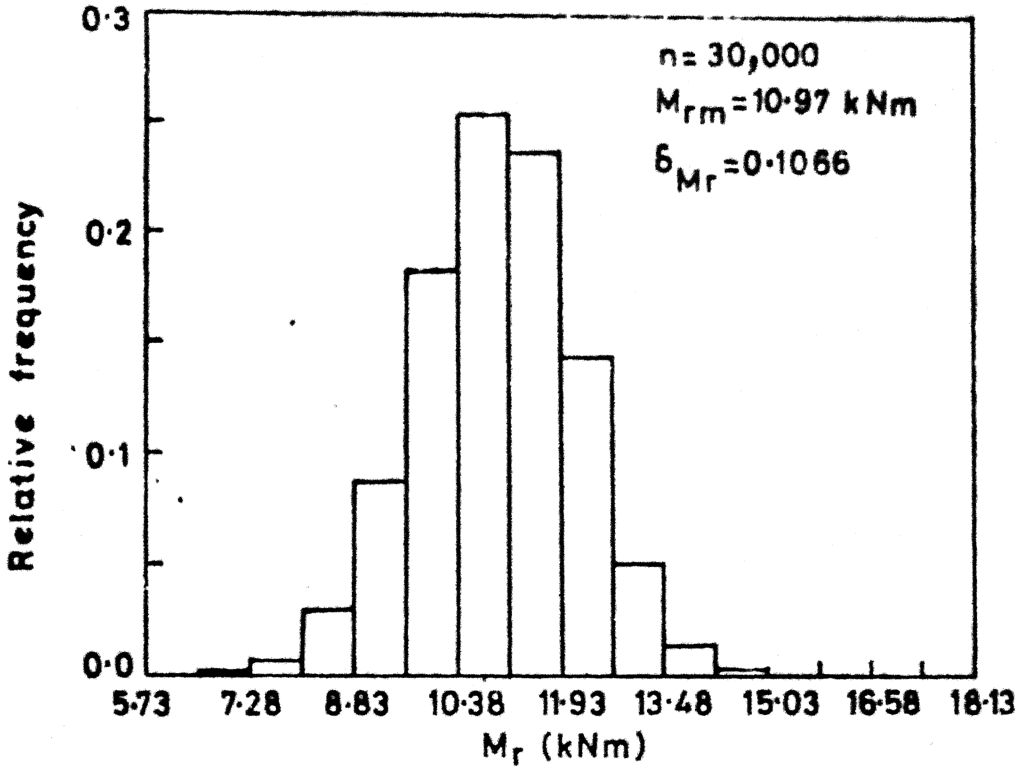
(b) Generated samples of f_y N (449·15, 51·29)



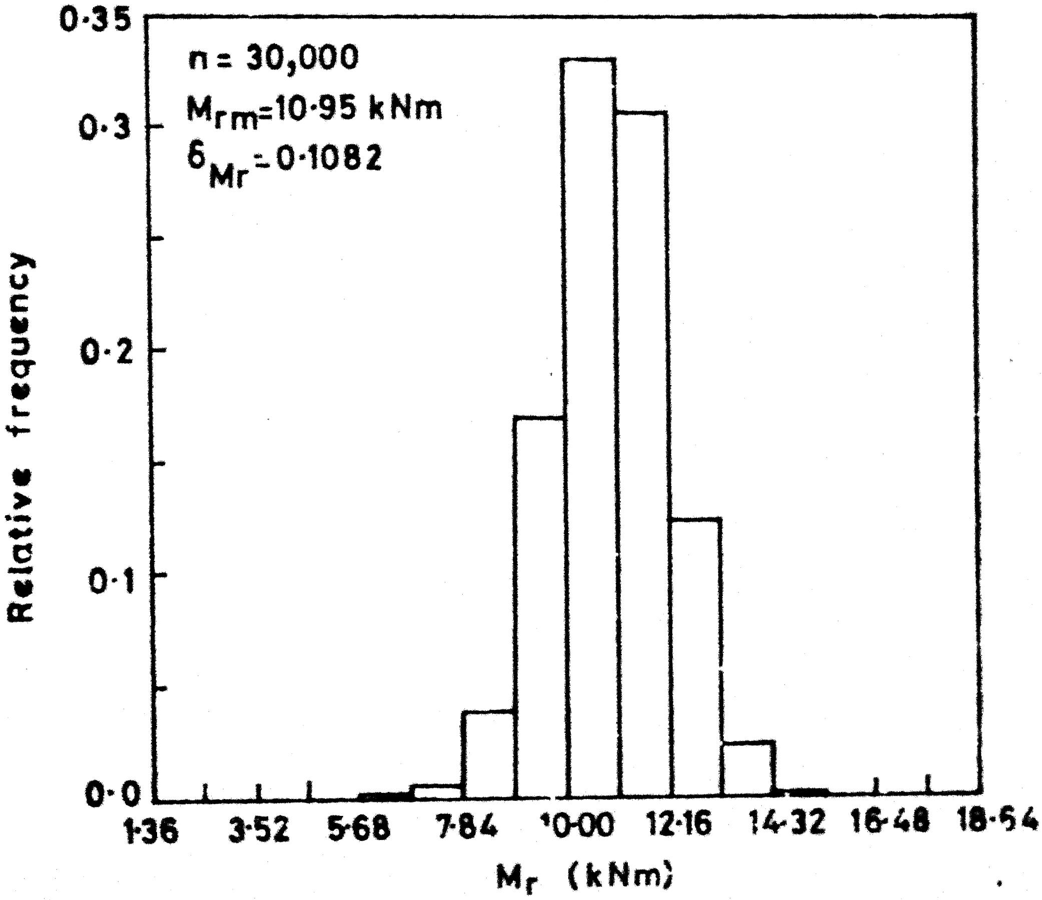
(c) Samples of M_r using generated samples of f_w and f_y for A_{st}=150·8 mm^2

Fig. 4·10 Random sample

Figs. 4.11(a) and 4.11(b). The coefficient of variation of M_r is found to be around 11 percent in both cases. For the same C.O.V. of steel strength (δ_{fy} = 0.1142), the shape of the histogram changes with the coefficient of variation of masonry strength (δ_{fw}). Usual chi-square test is conducted to fit the data. The samples shown in Fig. 4.11(a) is found to follow normal distribution at 0.5 percent significance level. As δ_{fw} is increased to 0.20, the shape of the histogram changes drastically as seen in Fig.4.11(b) and does not follow normal distribution at 0.5 percent significance level. Fig. 4.12 shows another typical histogram of simulated samples of M_r for δ_{fy} = 0.05 and δ_{fw}=0.20. It can be seen that the histogram is negatively skewed with a coefficient of variation as low as 5.18 percent. Normal, lognormal, beta, type I extremal (smallest) and Type III extremal (smallest) have not satisfied chi-square test for the generated data even at 0.1 percent significance level. Samples of M_r are generated using different combinations of δ_{fw} and δ_{fy} to study the shape of the frequency distribution. It is observed that for a fixed δ_{fy}, the distribution becomes more and more negatively skewed with increase in δ_{fw}. For highly under-reinforced sections, the coefficient of variation of steel governs the coefficient of variation of moment capacity and normal distribution can be accepted as an approximate model to represent the probability distribution if δ_{fy} and δ_{fw} are limited to 15 percent.



(a) $\delta_{fy} = 0{\cdot}1142$, $\delta_{fw} = 0{\cdot}1406$



(b) $\delta_{fy} = 0{\cdot}1142$, $\delta_{fw} = 0{\cdot}20$

Fig. 4·11 Histogram of simulated moment capacity for $A_{st} = 150{\cdot}8\ mm^2$

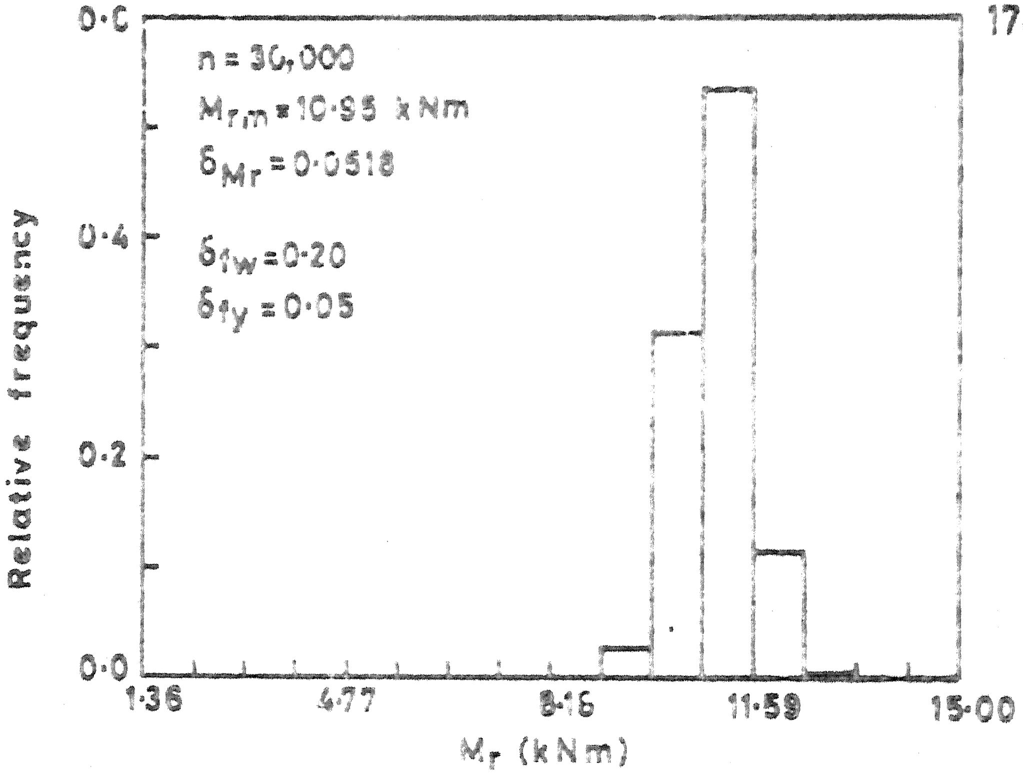


Fig. 4·12 Histogram of simulated samples of M_r for A_{st} = 150·8 mm^2

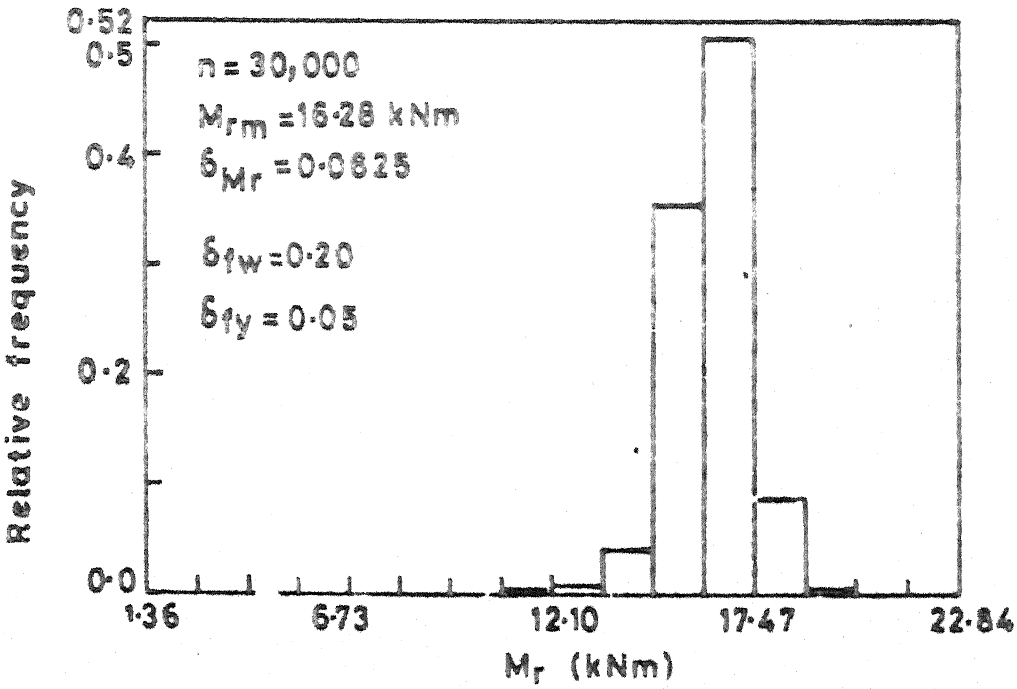


Fig. 4·13 Histogram of simulated samples of M_r for A_{st} = 235·62 mm^2

When the steel area is increased from 150.8 mm^2 (3 Nos. 8 Φ) to 235.62 mm^2 (3 Nos. 10 Φ), δ_{Mr} is found to increase from 5.18 percent to 6.25 percent for same δ_{fw}=0.20 and δ_{fy} = 0.05. The effect of increase in area of steel is shown in Fig. 4.13. The frequency distribution is found to be more negatively skewed and none of the common standard distributions satisfied chi-square test. Samples of M_r are generated with different steel area for different combinations of δ_{fw} and δ_{fy}. It is observed that as the area of steel is increased towards balanced steel area, the frequency distribution becomes more and more skewed to the right.

The samples of M_r as given in Fig. 4.13 are separated into under-reinforced and over-reinforced cases using the condition given in Eq. 4.11. The samples for which the section is under-reinforced and over-reinforced are separated and histograms are shown in Figs. 4.14 and 4.15 respectively. It can be seen that the frequency distribution of under-reinforced moment capacity M_{ru} has neglible skewness whereas the frequency distribution of over-reinforced moment capacity M_{ro} has a marked negative skewness. The coefficient of variation of M_{ru}($\delta_{M_{ru}}$) is found to be equal to 5.18 percent whereas the coefficient of variation of M_{ro}(δ_{Mro}) is found to be equal to 15 percent. Samples with different combinations of δ_{fw}, δ_{fy} and A_{st} are generated. It is found that the coefficient of variation of over-reinforced moment

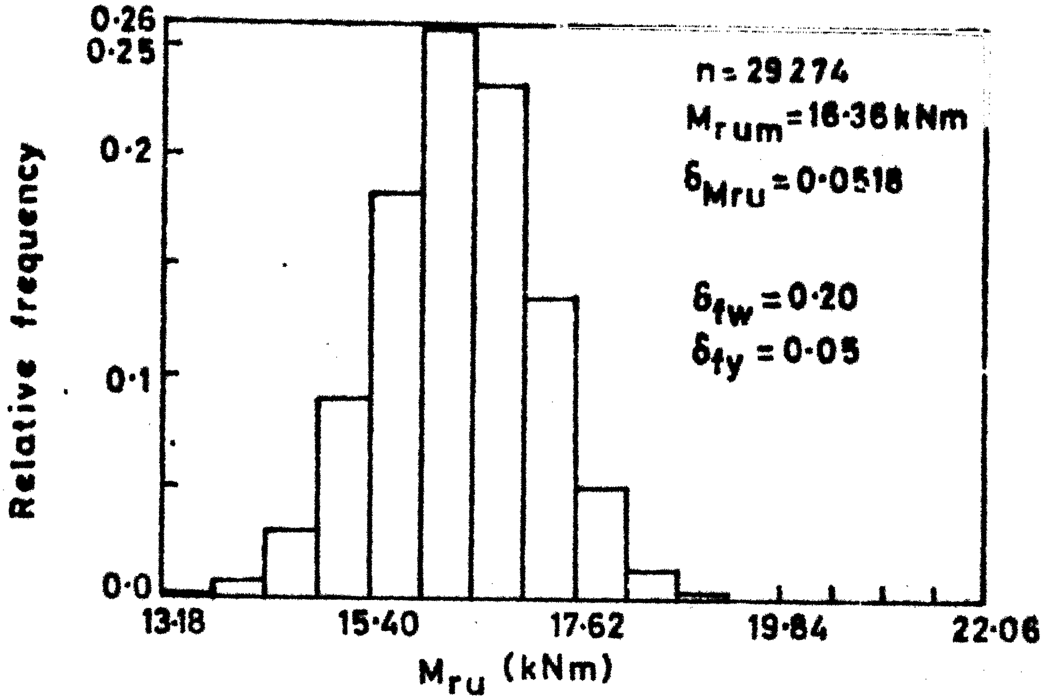


Fig. 4·14 Histogram of M_{ru} separated from M_r shown in Fig. 4·13

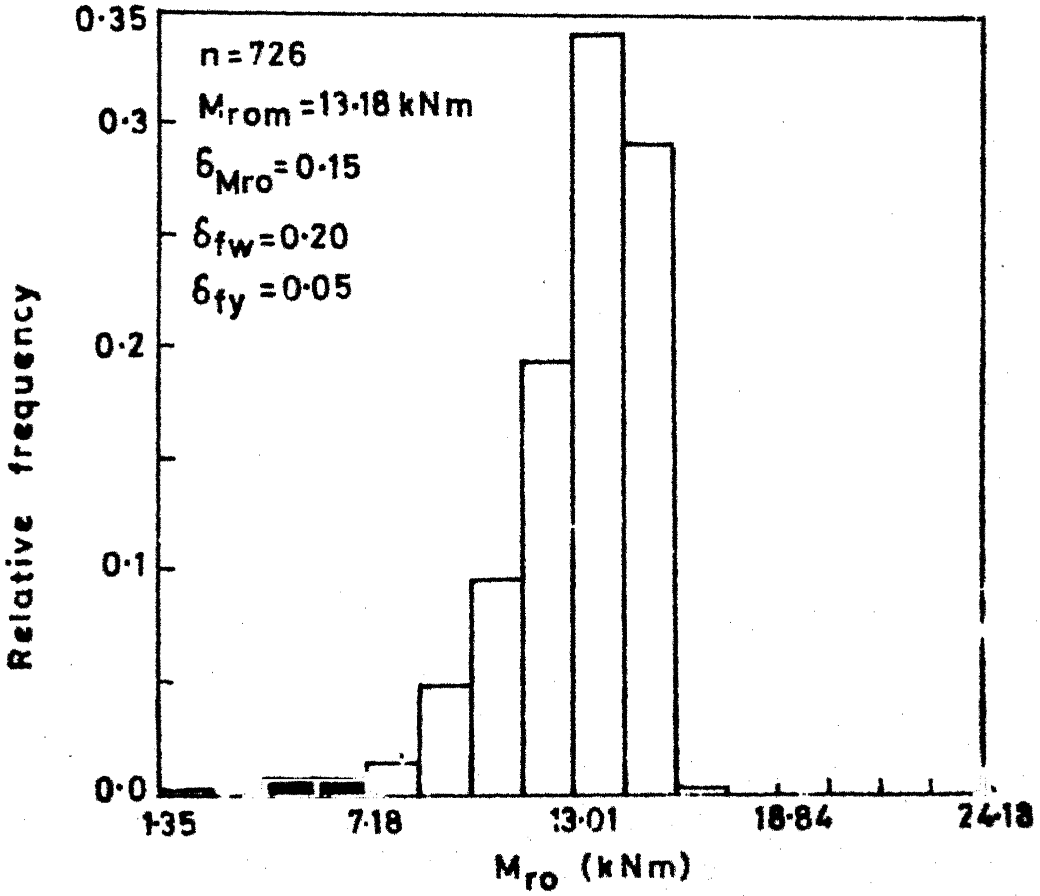


Fig. 4·15 Histogram of M_{rc} separated from M_r

capacity $\delta_{M_{ro}}$ is governed mostly by the C.O.V. of masonry strength δ_{fw} whereas the coefficient of variation of under-reinforced moment capacity $\delta_{M_{ru}}$ is governed mostly by the C.O.V. of steel strength δ_{fy}. The number of samples in over-reinforced case is found to be 726 out of 30000 sample as shown in Fig.4.15 which means that the probability of the section given in Fig. 4.4 with A_{st} = 235.62 mm^2 being over reinforced is equal to 0.0242. The probability of a section becoming over-reinforced is a function of δ_{fw}, δ_{fy} and area of steel A_{st}. It can be noticed from Figs. 4.14 and 4.15 that the mean of under-reinforced moment capacity M_{rum} is higher than that of over-reinforced moment capacity M_{rom} while the range of M_{ro} is higher than that of M_{ru}. Similar findings were observed in the simulated samples of prestressed concrete beams (85,86). This again depends on the relative magnitudes of δ_{fw}, δ_{fy} and area of steel provided.

Simulated samples of M_{ru} shown in Fig. 4.14 did not follow normal distribution at 0.5 percent significance level. Similarly, other distributions like lognormal, beta, Type I extremal (smallest), Type III extremal (smallest) etc. did not satisfy chi- square test to the samples of M_{ru}. Simulated samples of M_{ro} also did not follow any of the above distributions. Samples of M_{ru} and M_{ro} are generated for different combinations of δ_{fw} and δ_{fy}. It is observed

that samples of M_{ru} follow normal distribution at 2.5 percent significance level for δ_{fw}=0.20 and δ_{fy}=0.1142 whereas M_{ru} for δ_{fw}=0.20 and δ_{fy}=0.05 didnot fit normal distribution even at 0.5 percent significance level. Similarly, it is found that the samples of over-reinforced moment capacity M_{ro}, for δ_{fw}=0.20 and δ_{fy}=0.1142, followed Type III extremal (smallest) at 1 percent significance level. Shapes of the frequency distributions of M_{ru} and M_{ro} were found to change with the relative choice of δ_{fw}, δ_{fy} and A_{st}. Thus particular distribution like normal for under-reinforced moment capacity and Type III extremal (smallest) for over-reinforced moment capacity cannot always be accepted.

Probability of failure p_f is calculated from the simulated samples of M_r in the following way. The number of samples falling below M_e is computed and p_f is calculated as

$$p_f = P\ (M_r \leq M_e) \approx \frac{n_1}{n} \tag{4.48}$$

where

M_e = external moment (deterministic value)

n_1 = number of samples less than or equal to M_e

n = total number of samples generated.

Probability of failure computed from simulated samples by Eq. 4.48 for different values of M_e are shown in Table 4.3 and compared with the results obtained by analytical method through Eqs, 4.28, 4.40 and 4.42. It can be seen

Table 4.3 : Comparison of Simulated Results

$\frac{M_{rm}}{M_e}$	Probability of failure p_f for deterministic M_e							
	δ_{fy}=0.05				δ_{fy}=0.1142			
	δ_{fw}=0.1406		δ_{fw}=0.20		δ_{fw}=0.1406		δ_{fw}=0.20	
	Simulation	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation	Analytical
1.00	0.511	0.515	0.525	0.529	0.505	0.507	0.512	0.516
1.10	3.32(-2)	3.05(-2)	4.31(-2)	4.04(-2)	2.01(-1)	2.01(-1)	2.10(-1)	2.08(-1)
1.20	7.33(-4)	2.71(-4)	2.30(-3)	1.40(-3)	6.12(-2)	6.14(-2)	6.45(-2)	6.48(-2)
1.30	-*	2.14(-6)	8.33(-4)	4.90(-4)	1.74(-2)	1.64(-2)	1.90(-2)	1.78(-2)
1.40	-	5.74(-7)	5.00(-4)	3.13(-4)	4.60(-3)	4.21(-3)	5.43(-3)	4.75(-3)
1.50	-	2.66(-7)	3.67(-4)	2.11(-4)	1.07(-3)	1.09(-3)	1.57(-3)	1.36(-3)
1.60	-	1.33(-7)	3.33(-4)	1.48(-4)	4.33(-4)	2.92(-4)	9.00(-4)	4.55(-4)

* could not be found by simulation.

Note: f_{wm} = 8.96 N/mm^2 , f_{ym}=449.15 N/mm^2 and M_{rm}=10.989 kNm

Properties of the section are as shown in Fig. 4.4.

from Table 4.3 that the values of p_f by two different methods have very small difference. It is worthwhile to mention that minimum probability of failure computed by Eq. 4.48 is dependent on the number of samples generated. In this case, it is equal to 1/30000, i.e., 3.33×10^{-5}.

4.9 Discussions and Conclusions

Ultimate moment capacity of a RBB section is a function of geometric proportions, area of steel and strengths of masonry and steel. Due to random variations in strengths of masonry and steel, there exists a probability that the section will fail as an over-reinforced even though the section is designed as an under reinforced based on deterministic analysis. Probability of failure p_f of the section is the sum of the probabilities p_{fu} and p_{fo}. The computation of failure probabilities p_{fu} and p_{fo} involves evaluation of multiple integrals and depends on the probability distributions of masonry strength and steel strength. The probabilistic variations of dimensional properties of the section are not incorporated in the present formulation. The expressions for p_{fu} and p_{fo} are derived from the equations of moment capacity given by Eqs. 4.17 and 4.18 which are also valid for representing the moment capacity of singly reinforced RC and prestressed concrete sections. Thus, the expressions of p_{fu} and p_{fo} can be used for reinforced and prestressed concrete beam sections also.

The probabilities p_{fu} and p_{fo} are found to be sensitive to relative magnitudes of the coefficients of variation of strengths of masonry and steel and M_{rm}/M_e ratio. If the area of steel in a RBB section is increased, the probability of the section being over-reinforced increases. Consequently, under-reinforced failure probability (p_{fu}) decreases and over-reinforced failure probability (p_{fo}) increases but the probability of failure of the section (p_f) decreases for the same external moment. Therefore, if the area of steel actually provided is more than that required to achieve a particular reliability of the section, the design will be on safer side but the probability of brittle failure will increase. It is observed that p_f decreases to a limiting value equal to p_{fo} as the steel area is increased to a very large value. This signifies that as $A_{st} \to \infty$, the probability of failing as an under-reinforced is almost equal to zero for heavily over-reinforced sections. The situations where the probability of over-reinforced failure will govern the overall failure probability p_f, depends on the relative magnitudes of coefficients of variation of strengths of masonry and steel, A_{st}/A_{stb} ratio and M_{rm}/M_e ratio. For probabilistic external moment, the probability of failure increases as the coefficient of variation of external moment increases.

Samples giving ultimate moment capacity (M_r), are generated by Monte Carlo simulation using the randomly

generated data of strengths of masonry and steel. Histogram of the simulated samples has shown a marked negative skewness. As the percentage of steel area is increased to that of a balanced value, the distribution of M_r becomes more and more skewed to the right. The simulated sample has not followed any standard probability distribution. The samples of M_r are seperated into under-reinforced and over-reinforced cases. Normal and Type III extremal (smallest) distributions have been tried to fit the under-reinforced and over-reinforced samples respectively. Shapes of the frequency distributions of under-reinforced and over-reinforced samples change with the relative choice of coefficients of variation of strengths of masonry and steel, and also with the A_{st}/A_{stb} ratio. Although normal and Type III extremal (smallest) distributions are found satisfactory to represent the under-reinforced and over-reinforced samples in some particular cases, but they cannot be accepted in general. For a highly under-reinforced section (deterministically), normal distribution is found to be acceptable to represent the moment capacity M_r.

CHAPTER 5

RELIABILITY BASED DESIGN OF REINFORCED BRICK BEAMS

5.1 General

Reliability based design procedure of reinforced brick beam section at ultimate flexural strengths is presented in this chapter. Methods of computation of probability of failure for deterministic and probabilistic external moment, considering the probabilistic variations of strengths of materials, are given in Chapter 4. Design of a RBB section for a specified probability of failure reduces to a problem of solving an integral equation to find the geometrical properties or the area of steel. The method is illustrated with examples. Semi-probabilistic limit state design method is also illustrated. A set of partial safety factors for different variabilities of load and strength is also presented.

5.2 Introduction

Design methods can be broadly classified into two different categories : (i) deterministic design and (ii) probabilistic design.

In the deterministic design method, all the design variables are treated as deterministic quantities. The safety of the structure is ensured by providing more resistance than the external forces. Working stress method and ultimate load design method are based on deterministic approach. In working stress method, the safety is ensured by limiting the actual stresses to a set of permissible or allowable stresses whereas in ultimate load design the safety is ensured through proper specification of load factors.

Loads acting on a structure are probabilistic rather than deterministic. Similarly, strength of materials such as concrete, masonry, steel etc. are random variables. All these design variables are treated as probabilistic in probabilistic design. The safety of the structure is expressed through a specified probability of survival (reliability) that the structure will not be unserviceable over a given period or throughout the expected life of the structure. Three degrees of sophistications in probabilistic design are possible considering the statistical variabilities in resistance and load(113). At the highest sophistication, termed as Level III, the probability distribution of all variables and necessary convolutions must be known or computed and the members are designed for an accepted probability of failure previously agreed upon by the profession. The reliability is measured

or specified by a safety index or reliability index β in a level II design criteria where the safety is checked at certain discrete points (i.e., at selected values of mean live load to dead load ratio) of the limit state equation(72, 75,113). Probability distributions are not required in Level II methods. Level I methods involve selection of only one set of load and resistance factors which can be applied to all designs regardless of mean live load to dead load ratio (113). Current design practices are of Level I type although Levels I and II can be made equivalent by allowing load and resistance factors to vary with different load ratios.

5.3 Reliability Based Design of RBB Section

5.3.1 Introduction

The probability of failure p_f of a RBB section, as seen in Chapter 4, is the sum of under-reinforced failure probability p_{fu} and over-reinforced failure probability p_{fo}. Both these probabilities are functions of mean values and coefficients of variations of masonry strenth and strength of reinforcement and their probability distributions. Apart from these, p_{fu} and p_{fo} are functions of dimensional properties of the section, area of steel and the equation which defines the section as under-reinforced or over-reinforced. For deterministic external moment M_e, probability of failure can be expressed as

$$p_f = p_{fu} + p_{fo} \tag{5.1}$$

where $p_{fu} = g_1(M_e, f_{wm}, \delta_{fw}, f_{ym}, \delta_{fy}, b, d, A_{st})$

$p_{fo} = g_2(M_e, f_{wm}, \delta_{fw}, f_{ym}, \delta_{fy}, b, d, A_{st})$

Probabilities p_{fu} and p_{fo} are also dependent on the type of probability distributions of strengths of the masonry and steel. For probabilistic external moment, p_{fu} and p_{fo} depend also on the type of probability distribution function of external moment (M_e) and its parameters.

In a given design problem, the risk level p_f is only specified. Given the distributions along with necessary parameters (like mean, standard deviation etc.) of M_e, f_w and f_y, area of steel A_{st} can be solved from Eq. 5.1 for an assumed set of values of b and d. Thus any set of b, d and A_{st}, which gives probability of failure less than the prescribed risk, is a feasible design. Breadth (b) and effective depth (d) of the section can be chosen from practical limitations and A_{st} can be solved for. As seen in Chapter 4, Eq. 5.1 is an integral equation. Design procedure for deterministic and probabilistic external moment is given in the following sections.

5.3.2 Reliability based design for probabilistic variations of strengths of materials and deterministic external moment

It is assumed here that the dimension of the section is given and the area of steel is to be determined for a given probability of failure. If the external bending moment is assumed as deterministic and strengthsof materials follow normal distribution, area of steel can be determined by solving the integral equation given by Eq. 4.28, 4.40 and 4.42 for assumed values of b and d. The method is illustrated through the following example.

<u>Example 5.1</u>

A reinforced brick beam section has to carry an external bending moment of 7 kNm. Design the section for probability of failure of 10^{-5}. Strength of masonry f_w and strength of steel f_y are distributed as N (8.96,1.26) N/mm^2 and N (449.15, 51.29) N/mm^2 respectively.

Given: $f_{wm} = 8.96$ N/mm^2 , $s_w = 1.26$ N/mm^2

$f_{ym} = 449.15$ N/mm^2, $s_y = 51.29$ N/mm^2

$M_e = 7$ kNm (deterministic value)

$p_f = 10^{-5}$

Breadth and effective depth of the section are assumed as:

$b = 350$ mm , $d = 175$ mm.

Substituting above values in Eqs. 4.28, 4.40 and 4.42 , area of steel A_{st} is found by iterations. The iteration is started with an initial value of A_{st} and the value of p_f is computed. Depending on whether the computed p_f is smaller or greater than the given probability of failure (i.e., $p_f = 10^{-5}$), A_{st} is decreased or increased correspondingly in the next iteration. Iteration is stopped when the difference between the computed p_f and given p_f is less than a specified tolerance. Using this procedure, A_{st} is found to be 182.47 mm^2. Mean moment capacity M_{rm} for $A_{st} = 182.47\ mm^2$, is equal to 13.076 kNm and load factor M_{rm}/M_e is equal to 1.87.

In this example the dimensions of the section are assumed and A_{st} is computed. It is possible that for an assumed set of b and d, the iterations may not converge and the computed p_f is always higher than given p_f even for an increasing value of A_{st}. This means that the section cannot be designed with those assumed dimensions of the section and accordingly either b or d has to be increased. The coefficients of variation in the example were given as $\delta_{fw} = 0.1406$ and $\delta_{fy} = 0.1142$.

For an assumed set of b and d, area of steel required for a specified probability is a function of δ_{fw}, δ_{fy} and M_e. Table 5.1 shows the effect of increase of δ_{fy} and M_e on the

Table 5.1 : Area of Steel Required for $p_f = 10^{-5}$, for Deterministic External Moment

M_e	$\delta_{fw}=0.1406, \delta_{fy}=0.05$			$\delta_{fw}=0.1406, \delta_{fy}=0.1142$		
	A_{st} (mm^2)	$\frac{M_{rm}}{M_e}$	A^*_{std} (mm^2)	A_{st} (mm^2)	$\frac{M_{rm}}{M_e}$	A^*_{std} (mm^2)
7 kNm	119.49	1.264	128.03	182.47	1.868	144.68
8 kNm	137.80	1.264	150.14	211.12	1.866	169.66
9 kNm	156.83	1.266	173.75	243.60	1.871	196.34

* A_{std}=area of steel by limit state of strength(see Appendix-B)

Note: $b = 350$ mm , $d = 175$ mm

$f_{wm} = 8.96$ N/mm^2 , $f_{ym} = 449.15$ N/mm^2

required A_{st}. It can be seen from Table 5.1, that the steel area increases with increase in M_e and δ_{fy} for same δ_{fw} as it is expected. The area of reinforcement by limit state of strength is also given in Table 5.1.

5.3.3 Reliability based design for probabilistic variations of strengths of materials and external moment

External bending moment M_e is assumed as lognormally distributed random variable. For f_w and f_y following normal distribution, and M_e following lognormal distribution, area of steel A_{st} can be found in a similar way as discussed in section 5.3.2 by solving the integral equation given by Eqs. 4.46, 4.28, 4.40 and 4.42 for assumed values of b and d of the section. The method is illustrated through the following example.

<u>Example 5.2</u>

Design a RBB section for $p_f = 10^{-5}$ when external moment M_e is lognormally distributed with mean 7 kNm and coefficient of variation 20 percent. Other parameters are same as in Example 5.1.

Given:

$$M_{em} = 7 \text{ kNm} \quad , \quad \delta_{M_e} = 0.20$$

$$f_{wm} = 8.96 \text{ N/mm}^2 \ , \ s_w = 1.26 \text{ N/mm}^2$$

$$f_{ym} = 449.15 \text{ N/mm}^2, s_y = 51.29 \text{ N/mm}^2$$

$$p_f = 10^{-5}.$$

Breadth and effective depth of the section are assumed as before

$$b = 350 \text{ mm} \;, \quad d = 175 \text{ mm}$$

M_e is lognormally distributed and the parameters of log-normal distribution are computed from Eqs. 4.44 and 4.45 and they are:

$$\sigma_{ln} = 0.198$$

$$M_{ln} = 6.864 \text{ kNm}$$

The solution of the integral equation is obtained by iteration as discussed earlier. For an assumed value of A_{st}, p_f is computed by Eq. 4.46 where $F_{M_r}(x)$ is given by Eqs. 4.28, 4.40 and 4.42. Comparing the computed p_f with given p_f, A_{st} is changed accordingly in the next iteration till the convergence is achieved. A_{st} is found to be 275 mm^2 and the corresponding mean moment capacity is 18.746 kNm. Central safety factor or load factor (M_{rm}/M_{em}) is found to be 2.678. Table 5.2 shows the effect of increasing δ_{M_e} and δ_{fy} on the required steel area for same δ_{fw}. It can be observed that for the same section, required A_{st} increases as the coefficient of variation of external moment increases which is expected. Load factor M_{rm}/M_{em} is more in case of probabilistic M_e than that of deterministic M_e and increases with coefficient of variation of external moment as seen in Tables 5.1 and 5.2. This is expected because as the statistical

Table 5.2 : Area of Steel Required for $p_f = 10^{-5}$, for Probabilistic External Moment

δ_{fy}	$\delta_{fw} = 0.1406$			
	$\delta_{M_e} = 0.20$		$\delta_{M_e} = 0.25$	
	A_{st} (mm^2)	$\frac{M_{rm}}{M_{em}}$	A_{st} (mm^2)	$\frac{M_{rm}}{M_{em}}$
0.0500	234.0	2.331	297.0	2.857
0.1142	275.0	2.678	347.0	3.244

Note: External moment is lognormally distributed with mean M_{em} = 7 kNm.

All other values are same as given in Table 5.1.

variation in external load increases, higher safety factor is needed for the same reliability.

External moment M_e acting on a section is equal to the sum of dead load moment M_D and live load moment M_L. Live load variation is significant compared to dead load variation. For small variation of dead load, dead load moment can be assumed as deterministic. Under this condition, external moment M_e can be written as

$$M_e = M_D + M_L \tag{5.2}$$

where M_D is deterministic value (i.e., constant) and M_L is probabilistic following probability density function $f_{M_L}(M_L)$. Assuming lognormal distribution for M_L, the probability density function of M_e can be written as

$$\begin{aligned} f_{M_e}(M_e) &= f_{M_L}(M_e - M_D) \\ &= \frac{1}{\sqrt{2\pi}\ \sigma_{Lln}(M_e - M_D)} . \exp\left[-\frac{1}{2}\left(\frac{\ln(M_e - M_D) - \ln M_{Lln}}{\sigma_{Lln}}\right)^2\right] \\ &\qquad M_D < M_e < \infty \end{aligned} \tag{5.3}$$

where M_{Lln} and σ_{Lln} are the parameters of lognormal distribution for live load moment M_L. Probability of failure p_f can be expressed using Eq. 4.36 and is given by

$$\begin{aligned} p_f &= \int_{M_D}^{\infty} F_{M_r}(M_e)\ f_{M_e}(M_e)\ dM_e \\ &= \int_{M_D}^{\infty} F_{M_r}(M_e) . \frac{1}{\sqrt{2\pi}\ \sigma_{Lln}(M_e - M_D)} . \exp\left[-\frac{1}{2}\left(\frac{\ln((M_e - M_D)/M_{Lln})}{\sigma_{Lln}}\right)^2\right] \\ &\qquad . dM_e \end{aligned} \tag{5.4}$$

Substituting

$$w = \frac{1}{\sqrt{2}} \cdot \frac{\ln((M_e - M_D)/\ M_{Lln}\ )}{\sigma_{Lln}}$$

Eq. 5.4 can be written in a simplified form as

$$p_f = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}}\ e^{-w^2} \cdot F_{M_r}(M_D + M_{Lln}\ e^{\sqrt{2}\ w\,\sigma_{Lln}}). \ dw \qquad (5.5)$$

Eq. 5.5 gives the probability of failure when external moment consists of deterministic dead load moment and lognormally distributed live load moment. For normal distributions of f_w and f_y, $F_{M_r}(x)$ has to be calculated by using Eqs. 4.28, 4.40 and 4.42. Parameters of lognormal distribution M_{Lln} and σ_{Lln} can be found from the mean and coefficient of variation of live load moment as usual. In an analysis problem, the parameters in the right handside of Eq. 5.5 are known and the integral can be computed by using Gauss-Hermite quadrature formula. On the otherhand, in a design problem, area of steel A_{st} can be determined by solving the integral equation given by Eq. 5.5 for an assumed set of b and d. The solution procedure is same as given in example 5.2.

5.4 Semi-probabilistic Limit State Design of RBB Section

5.4.1 Introduction

A reinforced brick beam section can be designed for a specified probability of failure if

the probability distributions of strengths of materials and load along with their parameters are known. This involves complicated procedure of solving integral equation which is not suitable for routine design in profession. Moreover, complete knowledge of the probability distributions of different variables are not known in real life. To avoid complicated design procedures, probability based deterministic code formats were suggested by many researchers (70,71,72,74,75). Limit state design methods can partly incorporate statistical variations in some of the variables in a deterministic way and may be called semi-probabilistic approaches. Probabilistic variations of strengths of materials and loads are taken into account through characteristic strengths and loads, and the safety is ensured by proper prescription of partial safety factors applied to strengths and loads for different limit states considered. Codes based on limit state format for reinforced concrete are being used in the profession (39,40,41). Code of practice based on limit state format for reinforced brick-work has come in a draft form (48) incorporating statistical variations in strengths of masonry and steel, and loads. Based on the formats suggested by others (70,71,72 ,74,75,113, 114), partial safety factors are arrived at using the results of statistical analysis.

5.4.2 Characteristic strengths of materials

Characteristic strength of a material is defined as the strength below which the test results are expected to fall with an agreed probability. It is given by

$$R_k = R_m - k\,s \qquad (5.6)$$

where R_k = characteristic strength of material

R_m = mean strength

s = standard deviation

k = a factor which defines the acceptable risk.

BS 5628: Part 1, 1978(43) defines characteristic strength of masonry as the 'strength below which the probability of test results falling is not more than 5 percent'. Assuming normal distribution with accepted risk of 5 percent, value of k works out to be 1.645(111). Thus characteristic strength of masonry f_{wk} is given as

$$f_{wk} = f_{wm} - 1.645\ s_w$$

$$= f_{wm}\ (1 - 1.645\ \delta_{fw}) \qquad (5.7)$$

where f_{wm}, s_w and δ_{fw} are mean, standard deviation and coefficient of variation of strength of masonry respectively. The CEB-FIP Committee(82) recommends 5 percent accepted probability of test results falling below the characteristic value for concrete and steel. Thus characteristic yield or proof strength of steel assuming normal distribution is given by

$$f_{yk} = f_{ym} - 1.645\ s_y$$
$$= f_{ym}(1 - 1.645\ \delta_{fy}) \qquad (5.8)$$

where f_{ym} , s_y and δ_{fy} are mean, standard deviation and coefficient of variation of strength of steel respectively.

5.4.3 Characteristic loads

Characteristic load S_k is defined as the load which has a 95 percent probability of not being exceeded during the life of the structure (41,43). Since data is not available to express loads in statistical terms, the loads specified by I.S. Code (115) can be assumed as the characteristic load(41). Characteristic load S_k may be expressed as

$$S_k = S_m\ (1 + k\ \delta) \qquad (5.9)$$

where S_m = value of the most unfavourable load with a 50 percent probability of its being exceeded once in the expected life of the structure

δ = coefficient of variation of load

k = coefficient depending on agreed probability of loading greater than S_k.

Dead load on a structure may be assumed as normally distributed with small coefficient of variation for practical purpose. For an agreed probability of 5 percent that the dead load will be greater than characteristic dead load D_k, Eq.5.9 becomes

$$D_k = D_m\ (1+1.645\ \delta_D) \qquad (5.10)$$

where D_m = mean value of dead load

δ_D = coefficient of variation of dead load.

The frequency distribution of floor live load was found to follow lognormal distribution from a load survey conducted at I.I.T. Kanpur(110). Assuming live load follows lognormal distribution, characteristic live load L_k can be expressed as

$$P\ (L \leq L_k) = \phi\ \left(\frac{\ln(L_k/L_{ln})}{\sigma_{ln}}\right) = 0.95 \tag{5.11}$$

where L_{ln} and σ_{ln} are the parameters of lognormal distribution. Using the relations between parameters of lognormal distribution with mean and coefficient of variation, Eq. 5.11 can be written as

$$L_k = \frac{L_m}{\sqrt{1+\delta_L^2}} \exp\left[\,1.645\ \sqrt{(\ln(1+\delta_L^2))}\,\right] \tag{5.12}$$

where L_m = mean value of live load

δ_L = coefficient of variation of live load.

Eq. 5.12 gives the relation between the characteristic live load with the mean live load.

5.4.4 Partial safety factors

Probability of failure of a RBB section can be stated as

$$p_f = P(M_r \leq M_e) = P(M_r - M_e \leq 0) \tag{5.13}$$

where M_r and M_e are moment capacity and external moment respectively. If the probability distribution of the safety margin M_r-M_e is known, p_f can be easily calculated. The

distribution of (M_r-M_e) is important in the calculation of probability of failure and not the distribution of other variables which affect the safety margin M_r-M_e(70). Structural design is generally done for low probability of failure (i.e., $p_f \leq 10^{-5}$ say). At very low risk level (for $p_f \leq 10^{-5}$), the calculated p_f is sensitive to the distribution of the safety margin. Regardless of the distribution of (M_r-M_e) and for independent M_r and M_e, probability of failure can be written as (70,72)

$$p_f = F_X \left(- \frac{M_{rm} - M_{em}}{\sqrt{s_{M_r}^2 + s_{M_e}^2}} \right) \qquad (5.14)$$

where $F_X(x)$ is the distribution function of the normalized random variable X defined by

$$X = \frac{(M_r-M_e) - (M_{rm} - M_{em})}{\sqrt{s_{M_r}^2 + s_{M_e}^2}} \qquad (5.15)$$

where M_{rm}, M_{em} = mean of M_r and M_e respectively

s_{M_r}, s_{M_e} = standard deviation of M_r and M_e respectively.

Eq. 5.14 can be written as

$$M_{rm} = M_{em} - F_X^{-1} (p_f) . \sqrt{(s_{M_r}^2 + s_{M_e}^2)} \qquad (5.16)$$

where $F_X^{-1}(.)$ is the inverse function of $F_X(.)$.

Assuming the distribution of the safety margin (M_r-M_e) follows normal distribution, the distribution function $F_X(x)$ becomes

standardized normal distribution function and Eq.5.16 becomes

$$M_{rm} = M_{em} + \beta \sqrt{(s_{M_r}^2 + s_{M_e}^2)} \qquad (5.17)$$

where $\beta = -\phi^{-1}(p_f)$

$= \phi^{-1}(1-p_f)$ for $p_f < 0.5$.

The safety index β relates mean values of moment capacity and external moment in terms of standard deviation of the safety margin. For normal distribution, β is related to the probability of failure as

p_f	β
10^{-3}	3.09
10^{-4}	3.72
10^{-5}	4.26
10^{-6}	4.75

To ensure that the probability of failure will be less than p_f, Eq. 5.17 takes the form of following design inequality :

$$M_{rm} \geq M_{em} + \beta \sqrt{(s_{M_r}^2 + s_{M_e}^2)} \qquad (5.18)$$

If the distribution of $M_r - M_e$ is not normal, Eq. 5.16 can still be expressed as Eq. 5.18 with β defined as $\beta = -F_X^{-1}(p_f)$. Defining a quantity α_1 as

$$\alpha_1 = \frac{\sqrt{(s_{M_r}^2 + s_{M_e}^2)}}{s_{M_r} + s_{M_e}} \qquad (5.19)$$

Eq. 5.18 takes the following form

$$M_{rm} \geq M_{em} + \beta\alpha_1 \; (s_{M_r} + s_{M_e}) \tag{5.20}$$

or $$M_{rm}(1-\beta\alpha_1 \; \delta_{M_r}) \geq M_{em}(1 + \beta\alpha_1 \; \delta_{M_e})$$

The central safety factor required for the design is given by

$$\nu = \frac{M_{rm}}{M_{em}} = \frac{(1+ \beta\alpha_1 \; \delta_{M_e})}{(1-\beta\alpha_1 \; \delta_{M_r})} \tag{5.21}$$

The design inequality given by Eq. 5.20 can be expressed in the convenient form

$$\gamma \; M_{rm} \geq F \; M_{em} \tag{5.22}$$

where $\gamma = (1 - \beta\alpha_1 \; \delta_{M_r})$

$F = (1 + \beta\alpha_1 \; \delta_{M_e})$

The coefficients γ and F are the strength reduction factor and load factor applied to the respective mean values. The two sides of the design inequality given by Eq. 5.22 are coupled by α_1. Value of α_1 given in Eq. 5.19 depends on the relative values of s_{M_r} and s_{M_e}. For any combination of values of s_{M_r} and s_{M_e}, value of α_1 lies between 0.707 to 1.0. For a practical range of s_{M_r} and s_{M_e}, value of α_1 was found to be 0.75 by minimizing the error in the linearization given by Eq. 5.19 (72,75). Value of α_1 that minimizes the error depends on the range selected for s_{M_r}/s_{M_e} ratio. Assuming

$\alpha_1 = 0.75$, the resistance and load side of Eq. 5.22 becomes independent of each other but interconnected by β. Thus, γ and F in Eq. 5.22 becomes

$$\gamma = (1 - 0.75\ \beta\ \delta_{M_r})$$
$$F = (1 + 0.75\ \beta\ \delta_{M_e}) \tag{5.23}$$

External moment M_e can be expressed as the the sum of dead load moment M_D and live load moment M_L

$$M_e = M_D + M_L \tag{5.24}$$

The mean value and standard deviation of M_e is given by

$$M_{em} \approx M_{Dm} + M_{Lm}$$
$$s_{M_e} \approx \sqrt{(s_{M_D}^2 + s_{M_L}^2)} \tag{5.25}$$

where M_{Dm} , M_{Lm} = mean values of M_D and M_L respectively

s_{M_D} , s_{M_L} = standard deviations of M_D and M_L respectively.

Substituting $\alpha_1 = 0.75$, and using the relations given in Eq. 5.25, Eq. 5.20 becomes

$$M_{rm} \geq M_{Dm} + M_{Lm} + 0.75\ \beta\ (s_{M_r} + \sqrt{s_{M_D}^2 + s_{M_L}^2}\ ) \tag{5.26}$$

Defining another quantity α_2 as

$$\alpha_2 = \frac{\sqrt{s_{M_D}^2 + s_{M_L}^2}}{s_{M_D} + s_{M_L}} \tag{5.27}$$

Eq. 5.26 becomes

$$\gamma\ M_{rm} \geq F_D\ M_{Dm} + F_L\ M_{Lm} \tag{5.28}$$

where

$$\gamma = (1 - 0.75\ \beta\ \delta_{M_r}) \tag{5.29}$$

$$F_D = (1 + 0.75\ \beta\alpha_2\ \delta_{M_D}) \tag{5.30}$$

$$F_L = (1 + 0.75\ \beta\alpha_2\ \delta_{M_L}) \tag{5.31}$$

Eq. 5.28 gives general form of design inequality where F_D and F_L are the partial safety factors applied to the mean values of dead load and live load moments respectively and γ is the strength reduction factor to be applied to the mean value of moment capacity. Partial safety factors F_D and F_L are coupled by α_2 which is given in Eq. 5.27. The coefficient α_2 given by Eq. 5.27 can be written in a convenient form as

$$\alpha_2 = \frac{\sqrt{\delta_{M_D}^2 + \delta_{M_L}^2 (M_{Lm}/M_{Dm})^2}}{\delta_{M_D} + \delta_{M_L}\ (M_{Lm}/M_{Dm})} \tag{5.32}$$

As seen in the above equation, α_2 depends on the magnitudes of δ_{M_D}, δ_{M_L} and M_{Lm}/M_{Dm} ratio. Effect of mean dead load moment to mean live load moment ratio, for different values of δ_{M_L} and β, on partial safety factors F_D and F_L are shown in Figs. 5.1 and 5.2. Range of ratio of mean live load moment to mean dead load moment is selected as 0.5 to 2.5. Figs. 5.1 and 5.2 are plotted using Eqs. 5.30 to 5.32 for coefficient of variation of dead load moment $\delta_{M_D} = 0.10$. It can be seen from Figs. 5.1 and 5.2 that the safety factor F_L,

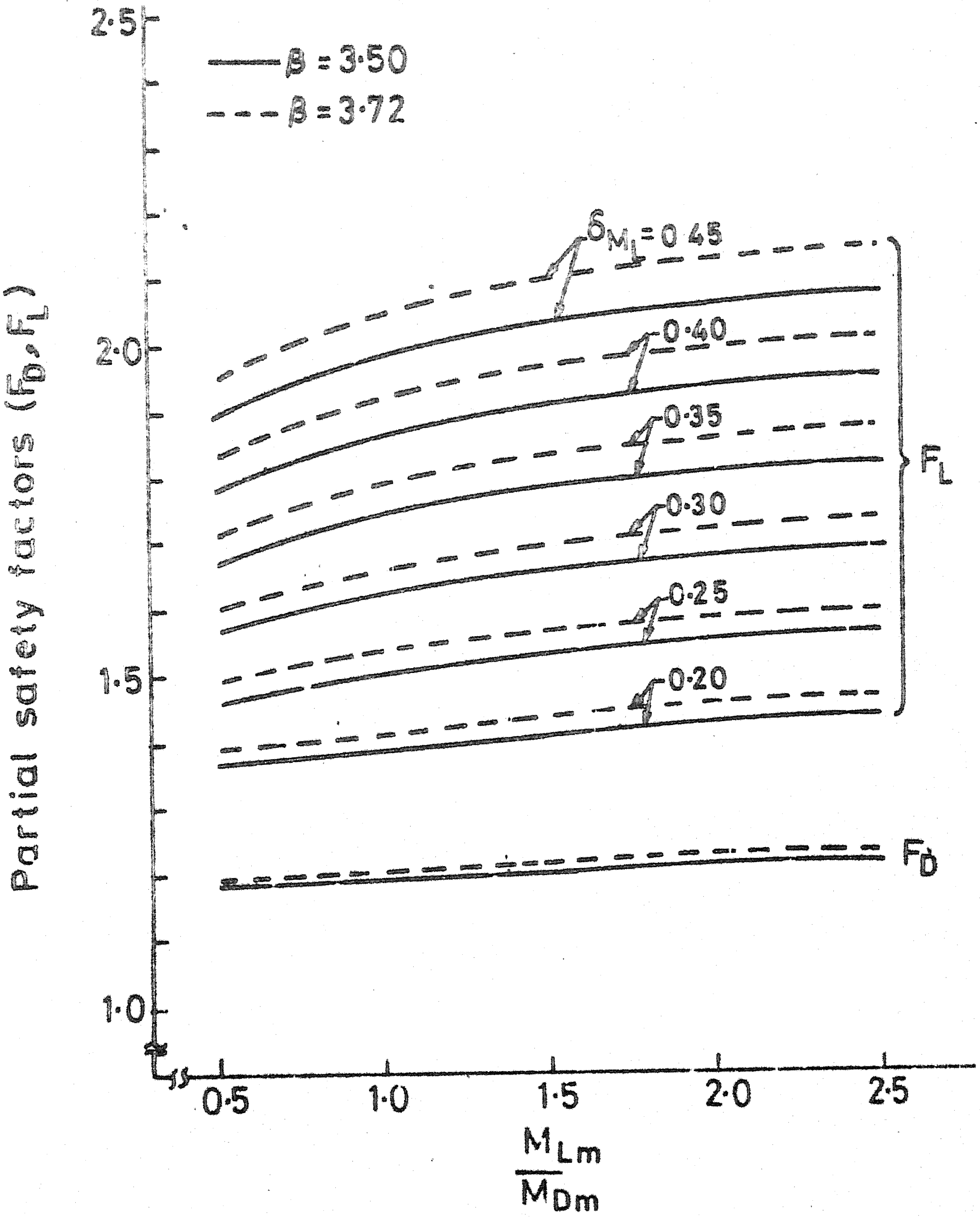


Fig.5·1 Variation of partial safety factors with M_{Lm}/M_{Dm} ratio

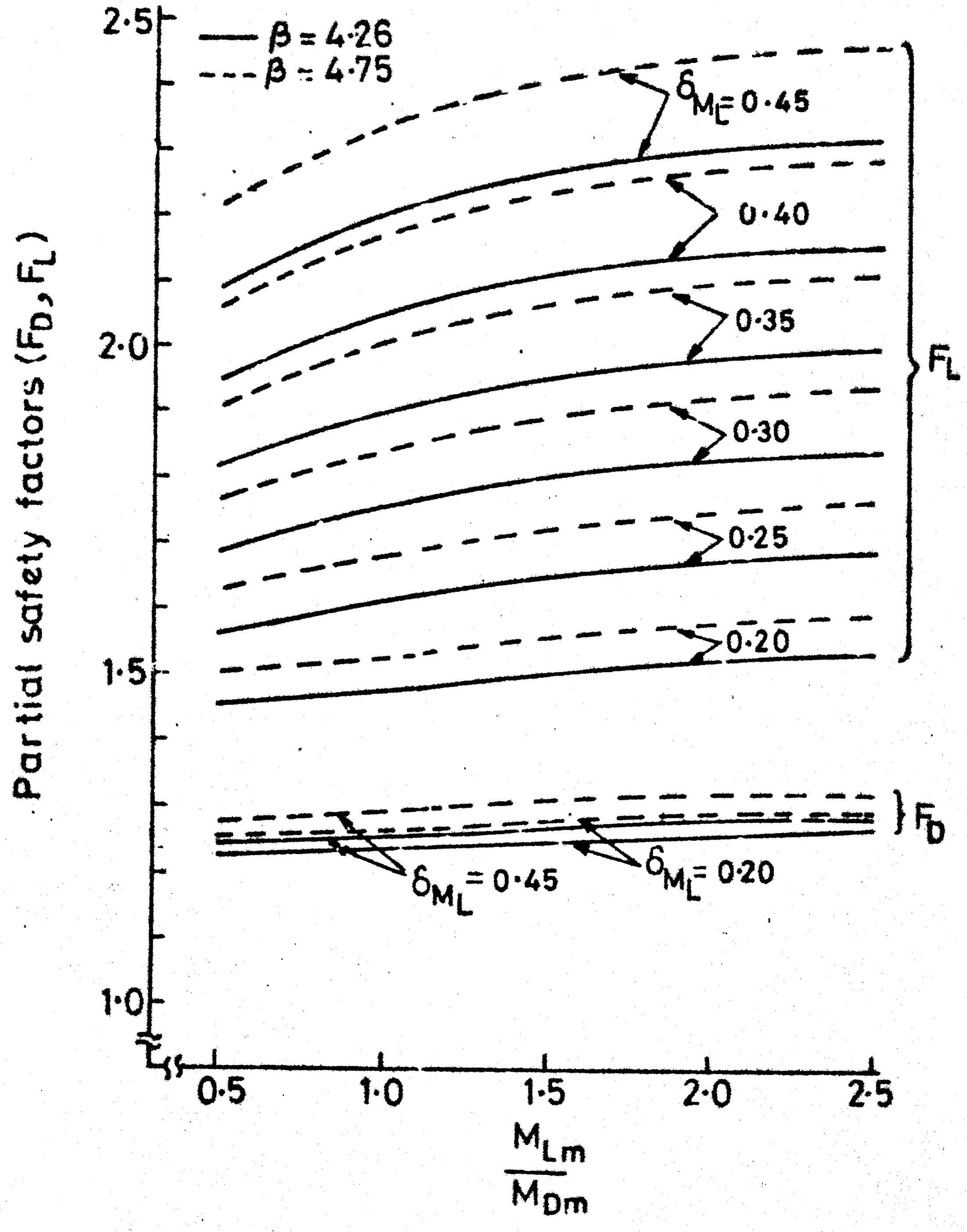


Fig.5·2 Variation of partial safety factors with M_{Lm}/M_{Dm} ratio

required to achieve a particular reliability (i.e., for constant β), increases with M_{Lm}/M_{Dm} ratio and also with coefficient of variation of live load moment δ_{M_L}. Since partial safety factor F_D is coupled to F_L by α_2, a slight increase is observed with increase in δ_{M_L} and M_{Lm}/M_{Dm} ratio. For a particular value of β, the change in F_D is insensitive to δ_{M_L} and M_{Lm}/M_{Dm} ratio as can be seen from Figs. 5.1 and 5.2.

Variation of strength reduction factor γ with coefficient of variation of moment capacity δ_{M_r} is shown in Fig. 5.3 for different values of reliability index β. Strength reduction factor γ decreases linearly as δ_{M_r} increases for a particular value of β. As the risk level decreases (i.e., β increases), strength reduction factor γ decreases for any particular value of δ_{M_r} as can be seen from Fig. 5.3.

From the strength reduction factor γ, the partial safety factor to be applied to strength of masonry is derived as follows. The design moment of an under-reinforced section can be written as

$$M_d = A_{st} \cdot \frac{f_{yk}}{\gamma_y} \cdot d\left(1 - 0.59\,\frac{A_{st}}{bd} \cdot \frac{f_{yk}/\gamma_y}{f_{wk}/\gamma_w}\right) \qquad (5.33)$$

where f_{yk} = characteristic strength of steel

f_{wk} = characteristic strength of masonry

γ_y = partial safety factor applied to steel

γ_w = partial safety factor applied to masonry.

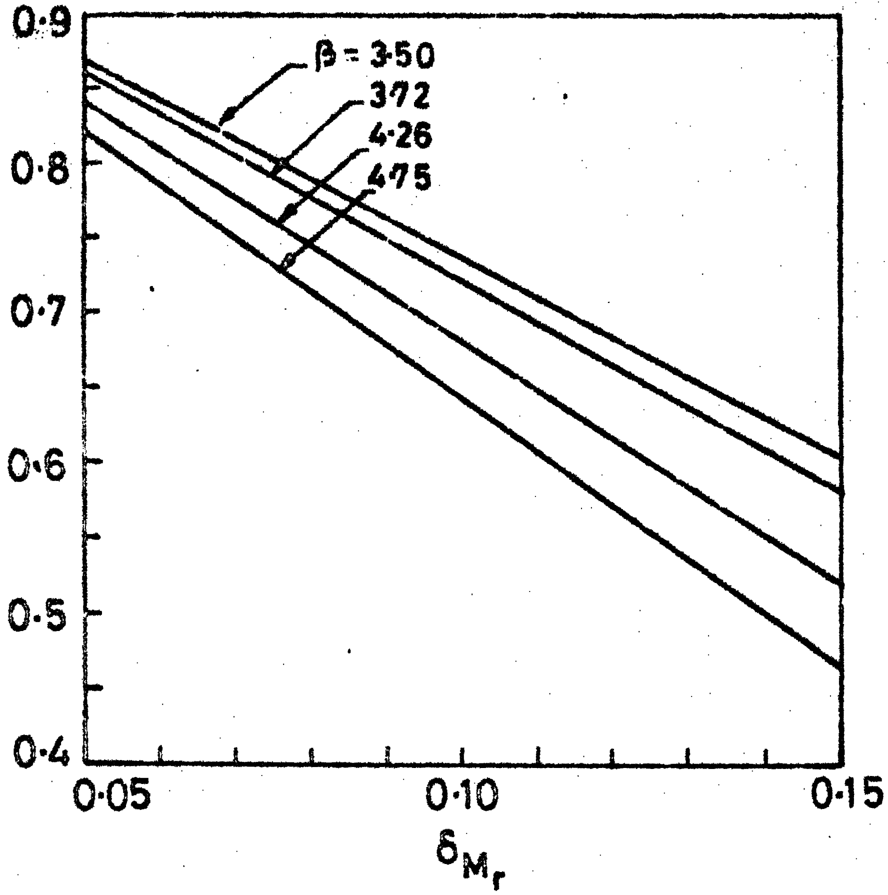


Fig.5·3 Strength reduction factor γ.

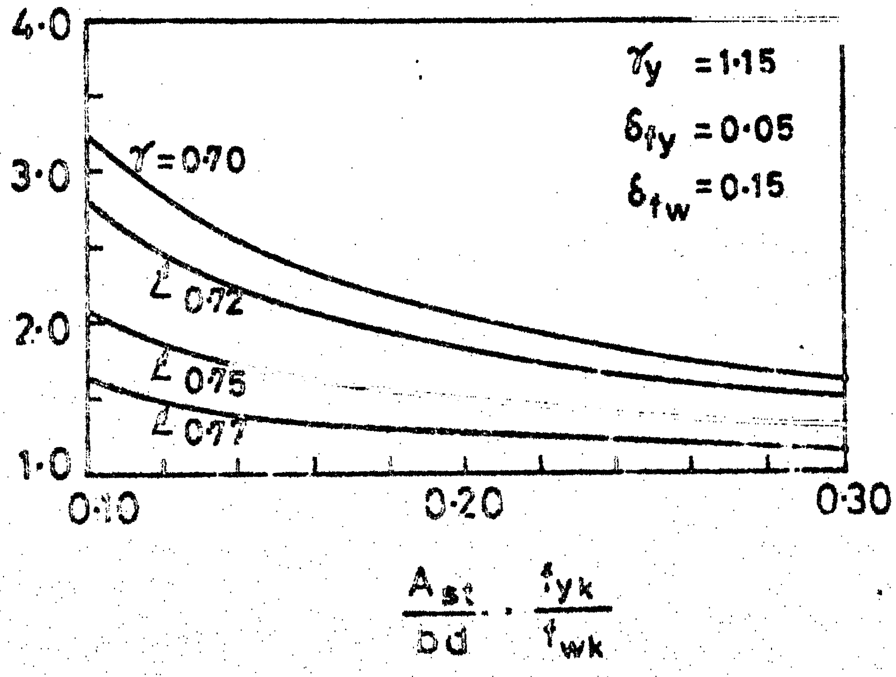


Fig.5·4 Partial safety factor for masonry

Substituting mean values of f_w and f_y in Eq. 4.11, the mean moment capacity is given by

$$M_{rm} = A_{st} f_{ym} d \left(1 - 0.59 \frac{A_{st}}{bd} \cdot \frac{f_{ym}}{f_{wm}}\right) \tag{5.34}$$

Since γM_{rm} is the design moment, partial safety factor γ_w can be found by equating M_d and γM_{rm}

$$M_d = \gamma M_{rm} \tag{5.35}$$

Substituting M_d and M_{rm} given by Eqs. 5.33 and 5.34 respectively in the above equation and using the following relations between characteristic and mean strengths

$$f_{yk} = (1 - 1.645\ \delta_{fy})\ f_{ym} = C_1\ f_{ym} \tag{5.36}$$

$$f_{wk} = (1 - 1.645\ \delta_{fw})\ f_{wm} = C_2\ f_{wm} \tag{5.37}$$

Partial safety factor γ_w is given by

$$\gamma_w = \frac{\gamma_y}{0.59\ C_3} \left(1 - \frac{\gamma\gamma_y}{C_1} \left(1 - 0.59\ C_3 \frac{C_2}{C_1}\right)\right) \tag{5.38}$$

where $C_3 = \dfrac{A_{st}}{bd} \cdot \dfrac{f_{yk}}{f_{wk}}$

Partial safety factor for steel (γ_y) is established as 1.15. Fig. 5.4 shows the plot of γ_w computed from Eq. 5.38 for different values of $\dfrac{A_{st}}{bd}\ \dfrac{f_{yk}}{f_{wk}}$ and strength reduction factor γ . Coefficients of variation of strengths of masonry and steel are chosen as 15 and 5 percent respectively. For other combinations of coefficients of variation of strengths of masonry and steel, γ_w is found to change

marginally and is not reported here.

The value of reliability index β is 4.26 for an accepted probability of failure of 10^{-5}. The coefficient of variation of dead load moment δ_{M_D} is chosen as 0.10. Partial safety factors F_D and F_L depend on β, the coefficient of variation of live load moment δ_{M_L} and the ratio of the mean values of the live to dead load moments. For $\beta = 4.26$, $\delta_{M_D} = 0.10$ and $\delta_{M_L} = 0.30$, partial safety factors F_D and F_L are selected from Fig. 5.2 as 1.3 and 1.8 respectively. The coefficient of variation of moment capacity δ_{M_r} depends on coefficients of variation of strengths of masonry and steel and percentage of reinforcement. δ_{M_r} is expected to be in the range of 0.05 to 0.10. For $\beta = 4.26$, the strength reduction factor γ is found to vary from 0.68 to 0.84 as seen from Fig. 5.3 for $\delta_{M_r} = 0.05$ to 0.10. Therefore, γ is selected as 0.7. In RBB, $\frac{A_{st}}{bd}$ is usually less than 0.5 percent. For $f_{yk} = 415$ N/mm^2 and f_{wk} in the range of 8 to 15 N/mm^2, $\frac{A_{st}}{bd} \cdot \frac{f_{yk}}{f_{wk}}$ varies in the range of 0.138 to 0.26. In this range, the partial safety factor for masonry strength γ_w is found to be 1.75 to 2.5 for $\gamma = 0.7$ from Fig. 5.4. If $\frac{A_{st}}{bd} \cdot \frac{f_{yk}}{f_{wk}} = 0.2$ is taken, γ_w is equal to 2.0 as can be seen from Fig. 5.4. BS 5628: Part 2 original draft code (48) has recommended partial safety factor for masonry equal to

2.5 to 2.8. The revised draft code BS 5628: Part 2 has now suggested the partial safety for masonry γ_W as 2.0 to 2.3 (134). A partial safety factor of 2 to 2.5 for strength of masonry is recommended, when partial safety factors 1.3 and 1.8 are used for dead and live load moments (to be applied to the mean values) respectively.

CHAPTER 6

DURABILITY OF REINFORCED BRICK MASONRY

6.1 Introduction

Durability of a structural system is defined as the ability to endure against the actions of destructive environment. The destructive process could be of chemical, physical or mechanical origin. Chemical factors are the most influential in causing corrosion of building materials. Humidity with oxygen is an important factor for corrosion under normal building conditions. The Government of Bihar at the request of National Building Organisation carried out an extensive survey (117) covering around 250 houses and other buildings in the state. A considerable deterioration was observed in the structures, the age of which varied from 20 to 40 years. Similar deterioration was observed in the case of about 3000 houses of age varying from 30 to 34 years in New Delhi (117). Some of the major defects and type of deterioration found in reinforced brickwork are given below (117,118):

(i) Cracks were observed in most of the buildings of age varying from 20 to 25 years. The damage was found to be less for the roofs which were protected from direct exposure of sunlight.

(ii) Although cracks had appeared in most of the buildings after 20 to 25 years, there were some instances where the plaster had been replaced just 12 years after construction.

(iii) A network of cracks was observed in the bottom face of the roofs along the main reinforcements and along the distribution bars.

(iv) The reinforcements were severely attacked and a considerable volume of partially oxidized material of dark colour was observed. In a good number of cases, it was noticed that the reinforcements of 1/2'' to 3/8'' diameter were reduced to half their diameter.

Dayaratnam (119) presented some results on durability case studies of reinforced brick masonry structures. It was concluded that the corrosion of reinforcement appears to be the most important factor that governs the serviceability and strength of reinforced brickwork. Durability aspect of reinforced brickwork was discussed by Foster (3). Corrosion of reinforcement was found to be related to the depth of cover and even 40 years old reinforcement bars were little affected provided they were not touching the bricks having a cover of about 50 mm of dense brick and mortar. Spalling of mortar in the joints and splitting of the bricks in horizontal planes at the level of reinforcements were reported by Dayaratnam(119).

Durability estimation is rather difficult because many uncertain environmental factors enter into the problem. It is time consuming and a satisfactory conclusion may not be easily related to all possible circumstances. Rajagopalan et al. (120) reported results of an investigation into the causes of corrosion of reinforcements in reinforced concrete and reinforced brick constructions. Attempts have been made by different researchers to study the durability aspects of reinforced brick masonry through possible short term measurements and observations, but it does not give any quantitative assessment of durability itself. The electrochemical process of corrosion of steel embedded in concrete was discussed by Bǎzant (121). The time to cracking or spalling of concrete due to corrosion was formulated by Bažant (122) and is given by

$$t_{cr} = t_p + t_{cor} \tag{6.1}$$

where t_{cr} = time at which corrosion would produce cracking or spalling of the concrete cover

t_p = time of depassivation

t_{cor} = duration of steady state corrosion.

Duration of steady state corrosion t_{cor} was related to the rate of rusting and the increase in diameter of the reinforcement bar due to corrosion. Different possible causes of deterioration of reinforced masonry with particular reference to corrosion of reinforcements are reviewed and discussed

in the following sections. An attempt is made to estimate the time to cracking or spalling of masonry due to corrosion of reinforcement following similar formulation given by Bažant (122). Finally, reliability analysis considering the decrease in moment capacity of reinforced brick beams with time for probalistic external moment is presented. Moment capacity of RBB at a particular time is assumed as deterministic function of initial random strength.

6.2 Causes of Deterioration

Deterioration of reinforced brick masonry structure can be broadly classified into three aspects (i) chemical attack on the mortar and brick, (ii) corrosion of embedded reinforcement and (iii) mechanical wear and tear due to repeated loads.

6.2.1 Chemical attack

Mortar present in brick masonry may be affected due to the attack by different aggressive media like acids, salts with exchangable ions. The degree of attack depends on the strength and concentration of the aggressive medium. The cement present in mortar is attacked more easily. The main components of ordinary portland cement are (123):

(i)	tricalcium silicate	$3CaO.SiO_2$
(ii)	dicalcium silicate	$2CaO.SiO_2$

(iii) tricalcium aluminate $3CaO.Al_2O_3$

(iv) tetracalcium aluminoferrite $4CaO.Al_2O_3.Fe_2O_3$

(v) gypsum $CaSO_4.2H_2O$

The deterioration of cement mortar due to chemical attack can be either dissolving attack or swelling attack. Strong mineral acids such as sulphuric acid, hydrochloric acid and nitric acid dissolve all components of set cement whereas weak acids and many organic acids only form water soluble compounds present in the cement (124). The degree of acid attack is specified through range of pH values in German standard DIN 4030 (124). pH value less than 4.5 has a very strong attack, between 4.5 to 5.5 a strong attack and between 5.5 to 6.5 a weak attack. Calcium, aluminium and iron salts, and silica gel form when strong acids react with set cement(124). Some typical reactions of sulphuric acid with different cement compounds are:

$$3CaO.2SiO_2.3H_2O+3H_2SO_4 \rightarrow 3CaSO_4+ 2(SiO_2.3H_2O) \qquad (6.2)$$

$$3CaO.Al_2O_3.6H_2O+6H_2SO_4 \rightarrow 3(CaSO_4.2H_2O) + Al_2(SO_4)_3 + 6H_2O \qquad (6.3)$$

$$3CaO.Fe_2O_3.6H_2O+6H_2SO_4 \rightarrow 3(CaSO_4.2H_2O) + Fe_2(SO_4)_3 + 6H_2O \qquad (6.4)$$

The typical reaction of hydrochloric acid on calcium silicate hydrate is:

$$3CaO.2SiO_2.3H_2O+6HCl \rightarrow 3CaCl_2+2SiO_2+6H_2O \qquad (6.5)$$

Carbonic acid is capable of dissolving lime from cement mortar or concrete and the reaction occurs partly as free carbonic acid and partly from the maintenance of carbondioxide-limestone equilibrium (124). Calcium hydroxide reacts with carbondioxide and forms calcium carbonate ($CaCO_3$) which further reacts with CO_2 in presence of water and forms calcium bicarbonate:

$$Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O \qquad (6.6)$$

$$CaCO_3 + H_2O + CO_2 \rightarrow Ca(HCO_3)_2 \qquad (6.7)$$

This process is known as carbonation. $CaCO_3$ is sparingly soluble whereas $Ca(HCO_3)_2$ is highly soluble in water. Thus it is capable of dissolving lime from cement mortar. Due to carbonation, pH value of cement mortar or concrete reduces.

Sparingly soluble and voluminous reaction products may form inside the set cement during corrosive reactions and can exert pressure upon the surroundings. Calcium aluminate present in portland cement when attacked by sulphate ions can form sulphoaluminate hydrate ($3CaO.Al_2O_3.3CaSO_4.32H_2O$), also called ettringite (124,125):

$$3CaO.Al_2O_3 + 3(CaSO_4.2H_2O) + 26H_2O \rightarrow 3CaO.Al_2O_3.3CaSO_4.32H_2O \qquad (6.8)$$

Magnesium sulphate may also react with calcium aluminate hydrate to form ettringite (126):

$$2(3CaO.Al_2O_3.12H_2O) + 3(MgSO_4.7H_2O)$$

$$\rightarrow 3CaO.Al_2O_3.3CaSO_4.31H_2O + 2Al(OH)_3 + 3Mg(OH)_2 + 8H_2O \quad (6.9)$$

Ettringite ($3CaO.Al_2O_3.3CaSO_4.30\text{-}32H_2O$) crystalizes in the form of needle shaped crystal and has a very large volume compared to the components it replaces because of the introduction of huge quantity of water of crystallization(123,126). As the space it has to occupy is limited, the voluminous compound thus formed subjects to its surroundings an enormous pressure and therefore produces cracking and spalling. Evidences of sulphate attack on brick wall is reported by Benningfield (127). If sulphates are present in the aqueous phase for prolonged periods or arise from ground water in situations below damp proof course, sulphate attack is possible. Normal building bricks of low permeability and suction rate offer good resistance against aggressive medium and are absolutely resistant to chemical action from the atmosphere(124). Bricks must be free of swelling components which may cause flaking off or destructive leaching action.

6.2.2 Corrosion of reinforcement

Reinforcement is normally placed in mortar joints of reinforced brickwork. The environment surrounding the reinforcement changes with time. Before the construction, the steel bars are exposed to atmospheric rusting which is due to

the simultaneous presence of water and oxygen (air). At the time of construction, the bars are surrounded by freshly mixed cement mortar which is normally so alkaline that it prevents further corrosion of embedded steel. The protective role of the cement mortar depends on its ability to maintain an alkaline environment round the reinforcing steel and its impermeability to oxygen(air), moisture and other corrosive chemicals.

Corrosion of reinforcement is an electrochemical process. The likelihood of corrosion for a metal immersed in an electrolyte solution depends on the pH value of the solution and the electrical redox potential of the metal (128). Fig. 6.1 shows the simplified Pourbaix diagram of iron(129). This shows the equilibrium regions where the metal is in a state of immunity, passivity or where corrosion will occur. The Pourbaix diagram for carbon steel is essentially the same as that of iron shown in Fig. 6.1. In the pH interval of 9.5 to 12.5, as can be seen from Fig. 6.1, steel remains in a passive state. A layer of ferrous hydroxide forms on the metal surface which protects the steel from further corrosion. Hydration of Portland cement produces calcium and other hydroxides which causes the final alkalinity in excess of pH 12(124,128,130) and offers a natural protection against corrosion in cement mortar or concrete. Typical

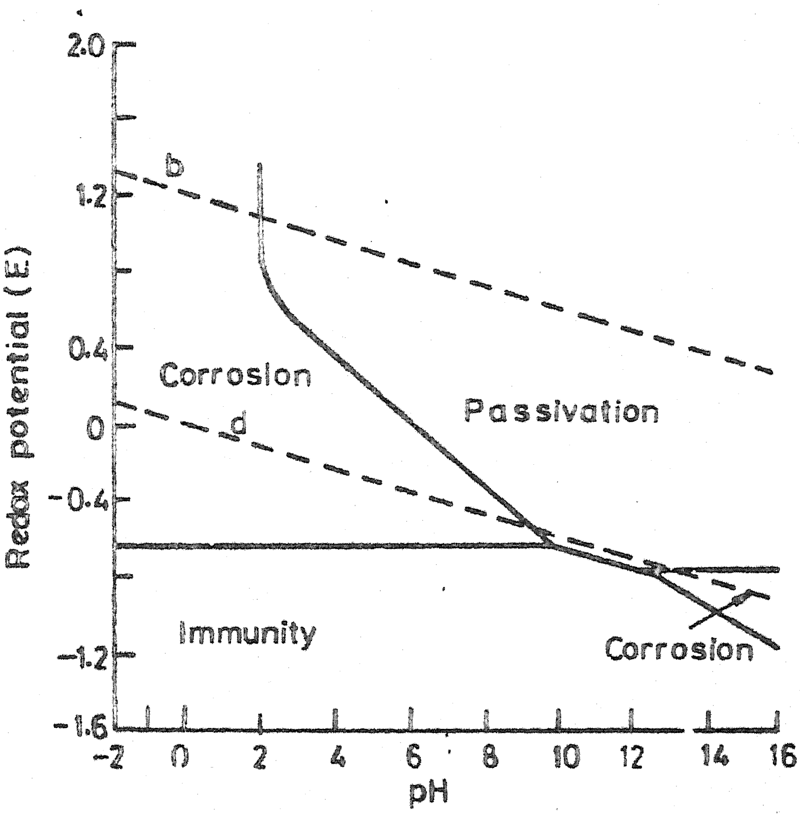


Fig. 6.1 Simplified Pourbaix diagram for iron (from ref. 129)

reactions of hydration of portland cement are (128)

$$2(3CaO.SiO_2)+6H_2O \rightarrow 3CaO.SiO_2.3H_2O+ 3Ca(OH)_2 \qquad (6.10)$$

$$2(2CaO.SiO_2)+4H_2O \rightarrow 3CaO.2SiO_2.3H_2O+Ca(OH)_2 \qquad (6.11)$$

Generally alkalinity of pore fluid of fresh cement mortar or concrete lies within pH 12 to 13 (130). Carbondioxide present in air dissolves in the moisture present in mortar to form weak carbonic acid which over a long period of time slowly reacts with the alkalis in the pore water of mortar. Thus, the pH may come down below 9.5 which no longer ensures passivity against corrosion. At constant temperature and average humidity the progress of carbonation is approximately proportional to the square root of time (124).

Simultaneous presence of water and oxygen is the main cause of corrosion of reinforcement. Normally anode areas develop on steel surface and iron ions pass into the solution at the anode surface after the depassivation of steel (121,131)

$$Fe \rightarrow Fe^{++} + 2\,e^- \quad \text{(at anode)} \qquad (6.12)$$

Adjacent to anode acts as cathode and dissolved oxygen in pore water reacts with incoming electrons to form hydroxyl ions in presence of water

$$O_2 + 2H_2O + 4e^- \rightarrow 4\,OH^- \text{ (at cathode)} \qquad (6.13)$$

Electric current is created in steel due to liberation of electrons. The negative hydroxyl ions liberated at cathode

arrive at anodic area and electrically neutralizes the dissolved Fe^{++} forming a solution of ferrous hydroxide.

$$Fe^{++} + 2\ OH^{-} \rightarrow Fe(OH)_2 \text{ at anode} \qquad (6.14)$$

The total cell reaction is

$$2Fe + 2H_2O + O_2 \rightarrow 2Fe(OH)_2 \qquad (6.15)$$

Ferrous hydroxide is unstable in oxygenated solution(131) and further reacts with available oxygen and water to form ferric hydroxide ($Fe(OH)_3$) which precipitates from the solution.

$$4Fe(OH)_2 + 2H_2O + O_2 \rightarrow 4Fe(OH)_3 \qquad (6.16)$$

This final product constitutes hydrated red rust ($FeO(OH)+H_2O$) and has a volume of approximately four times that of steel(121). There are other forms of rust, for example $HFeO(OH)$, $HFeO_2$, $FeSO_4$ and Fe_3O_4(black rust). The volume of black rust is approximately twice the volume of steel. The typical reaction is

$$3Fe + 8\ OH^{-} \rightarrow Fe_3O_4 + 8\ e^{-} + 4H_2O \qquad (6.17)$$

Generally, the tensile reinforcements in reinforced brick beams and slabs are subjected to tensile stresses under service condition. Thus the danger of corrosion increases because of several flexural cracks which allows moisture and oxygen to reach the reinforcements more easily.

6.2.3 Mechanical wear and tear due to repeated loads

Effect of repeated loads on RB beams were studied by Srivastava (1, 132). The increase in deflection due to repeated loads was rapid upto about 0.2 mega cycle and stabilized after about 0.5 mega cycle. Cumulative residual deflection of the order of 1/1300 to 1/900 of the span was observed under repeated loads. Experiments on RB slabs under repeated loads (2) showed that the ratio of recovery and the ratio of additional deflection to cumulative deflection damp out with more number of cycles which show ductile nature of RB slabs. The upper limit of repeated load was kept at about 80 percent of the ultimate load capacity of the beams in both experimental investigation(1,2,132). Due to limited experiments the decrease in ultimate strength of RB beams and slabs could not be related with magnitude and range of repeated loads and number of cycles. However, cracks in brick and mortar joints were observed after application of repeated loads. Propagation of cracks through mortar joints were also observed in some of the beams.

6.3 Cracking of Masonry due to Reinforcement Corrosion

A typical RBB section is shown in Fig.6.2(a). Let D be the original diameter of the reinforcing bar. Since the volume of rust is more than the volume of steel which is converted to rust, the average diameter increases and as a result exerts pressure to the surrounding brickwork. Let

M_{st} be the mass of steel converted to rust per unit length of the reinforcement and M_{rust} be the mass of rust thus produced. The increase in volume per unit length ΔV can be written as

$$\Delta V = \frac{M_{rust}}{\rho_{rust}} - \frac{M_{st}}{\rho_{st}} = (V_r - 1) \frac{M_{st}}{\rho_{st}} \tag{6.18}$$

where

ρ_{rust} = density of rust

ρ_{st} = density of steel = 7.85 gm/cm^3

$$V_r = \frac{M_{rust}}{M_{st}} \cdot \frac{\rho_{st}}{\rho_{rust}} \tag{6.19}$$

= volume expansion ratio which depends on the type of compound produced.

Let ΔD be the average increase in diameter and can be found from Eq. 6.18

$$\Delta V = \frac{\pi}{4} \cdot \left[(D + \Delta D)^2 - D^2 \right] = (V_r - 1) \cdot \frac{M_{st}}{\rho_{st}} \tag{6.20}$$

Assuming $\Delta D \ll D$, and neglecting higher orders of ΔD, Eq. 6.20 can be written as

$$\Delta D = (V_r - 1) \cdot \frac{2}{\pi D} \cdot \frac{M_{st}}{\rho_{st}} \tag{6.21}$$

Eq. 6.21 gives the relation between ΔD and the mass of steel M_{st} which is converted to rust. Let p_r be the radial pressure developed on the surrounding brickwork due to increase in diameter ΔD as shown in Fig. 6.2(b). Two possible modes of cracks can develop due to the corrosion of reinforcement as shown in Figs. 6.2(c) and 6.2(d). If spacing of the bars (s) is large compared to the diameter

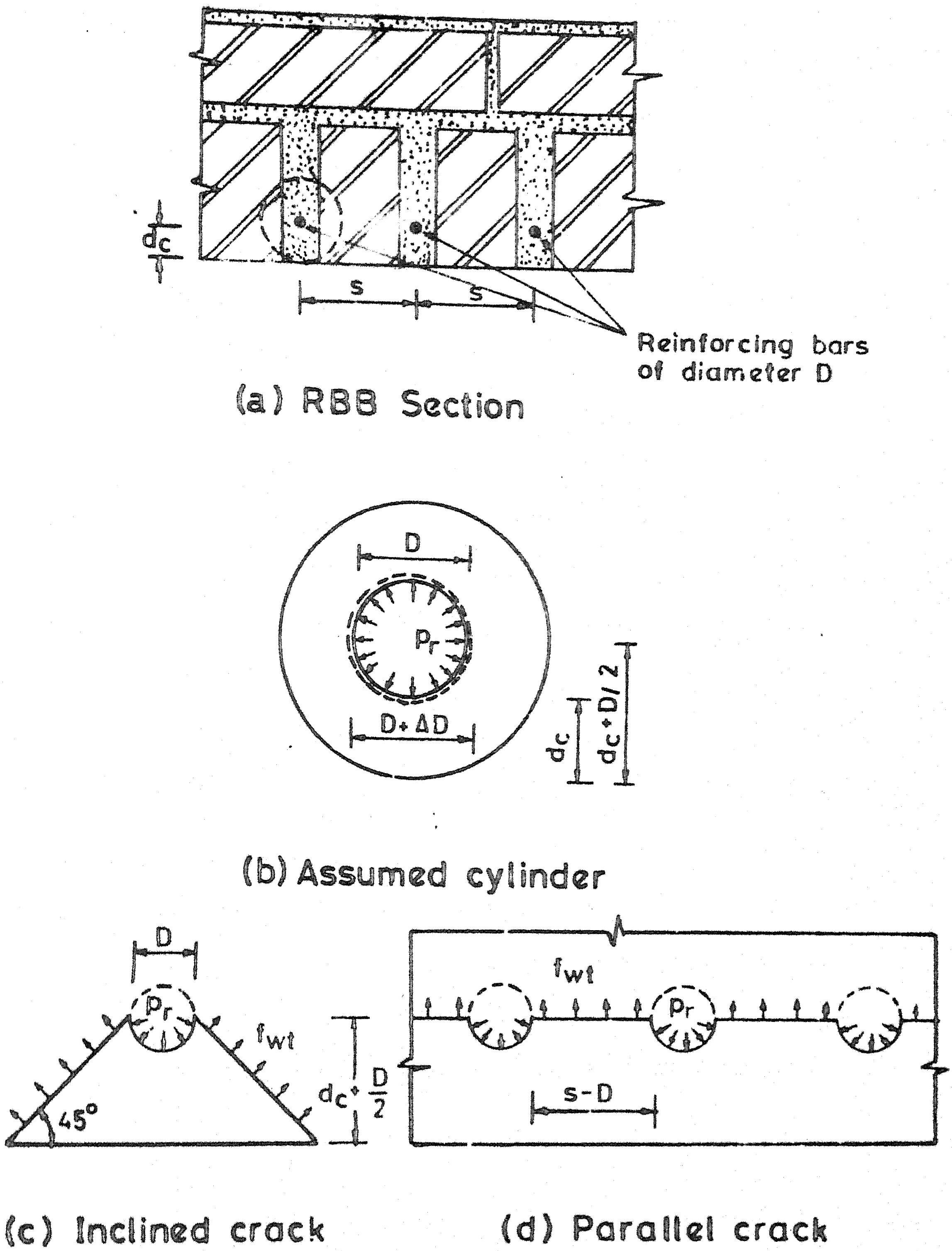


Fig. 6.2 Idealisation of the cracking pattern due to corrosion of reinforcement

of bar, failure mode consisting of 45^{o} inclined cracks can be assumed as shown in Fig. 6.2(c). Let f_{wt} be the average tensile strength of brickwork. Considering the equilibrium of vertical forces, p_r can be related to f_{wt} as

$$p_r D = 2 f_{wt} (d_c + \frac{D}{2}) \tag{6.22}$$

where d_c = clear cover to the reinforcement as shown in Fig. 6.2(c)

Inclined cracks of 45^{o} are found mostly in reinforced concrete constructions. If the spacing of bars(s) is not large compared to D and parallel bars rust simultaneously, parallel cracks as shown in Fig. 6.2(d) are expected which in turn may cause cover peeling. Splitting of the bricks in horizontal plane at the level of reinforcement at some places were observed (119). Equating the vertical forces, p_r can be related to f_{wt} as

$$p_r D = (s - D) f_{wt} \tag{6.23}$$

Although average tensile strength of brick work in parallel cracks will not be in general equal to the average tensile strength in inclined cracks, no distinction is made in the present formulation. The condition that a parallel crack will form is given by

$$d_c > \frac{s - 2D}{2} \tag{6.24}$$

Brickwork is not a homogeneous and isotropic medium. It is assumed here that the surrounding brickwork is homogeneous and isotropic medium for the following derivation. Assuming a thick cylinder of thickness d_c as shown in Fig. 6.2(b), ΔD can be related to the internal pressure p_r as

$$\Delta D = a_r \, p_r \tag{6.25}$$

where a_r is the flexibility coefficient which depends on Poisson's ratio, Young's modulus and other parameters. Under the assumption of plane strain, a_r is given by

$$a_r = \left[1 + \nu + \frac{(1 - \nu^2)\, D^2}{2d_c(d_c + D)} \right] \cdot \frac{D}{E} \tag{6.26}$$

where ν = Poisson's ratio

E = Young's modulus of brickwork

d_c = Thickness of assumed cylinder, i.e., clear cover to the reinforcement.

Assumption of plane strain is justified when the length of the member is large compared to the other dimensions and corrosion occurs in a considerable length. If there are cracks perpendicular to the reinforcements (as may be the case in RB beams at bending), plane stress assumption is justified. Under the assumption of plane stress, a_r is given by (122)

$$a_r = \left[1 + \nu + \frac{D^2}{2d_c(d_c + D)} \right] \cdot \frac{D}{E} \tag{6.27}$$

If a cylinder of infinite space ($d_c \to \infty$) is assumed,

$$a_r = \frac{(1+\nu)D}{E} \qquad (6.28)$$

Eq. 6.28 is valid under both plane strain and plane stress assumptions. Thus, coefficient a_r in Eq. 6.25 is bounded as

$$a_r' < a_r < a_r''$$

where a_r' is given by Eq. 6.28 and a_r'' is given by Eqs. 6.26, or 6.27 for plane strain or plane stress cases respectively. For practical purpose, the flexibility coefficient may be taken as

$$a_r = (a_r' + a_r'') / 2 \qquad (6.29)$$

When parallel bars rust, the effect of adjacent bars can be taken into account and an approximate formula is given by Bazant (122). For s > 10 D, the effect due to adjacent bars on a_r is around 2 percent and can be neglected. Substituting value of ΔD from Eq. 6.25, Eq. 6.22 and Eq. 6.23 become

$$\begin{aligned} \Delta D &= 2\, f_{wt}\, (d_c + \frac{D}{2}) \cdot \frac{a_r}{D} \quad \text{(inclined crack)} \\ &= f_{wt}\, (s-D) \cdot \frac{a_r}{D} \quad \text{(parallel crack)} \end{aligned} \qquad (6.30)$$

Eq. 6.30 gives the increase in diameter ΔD required to cause cracking. The mass of steel M_{st} which has to be converted to rust to cause cracking or spalling of brickwork is given by (from Eq. 6.21)

$$M_{st} = \frac{\rho_{st}\pi}{(V_r - 1)} \cdot f_{wt} \left(d_c + \frac{D}{2}\right) . \ a_r \quad \text{(for inclined crack)}$$

$$= \frac{\rho_{st}\pi}{2(V_r - 1)} \cdot f_{wt}(s-D) . \ a_r \quad \text{(for parallel crack)}$$

$$(6.31)$$

Assuming that the steady state corrosion begins after the depassivation time t_p, the time to cracking due to corrosion is given by Eq. 6.1, where $t_{cor} = M_{st}/ j_{cor}$ and j_{cor} is the rate at which the steel is converted to rust.

The above formulation is based on the assumption that there is no void or clearence between the bar and surrounding brickwork, i.e., a slight increase in diameter will start developing pressure to the surrounding medium. In a practical situation, a thin flexible film surrounding the reinforcement or any other type of void may exist and in that case t_{cr} can be estimated by modifying Eq. 6.1 as

$$t_{cr} = t_p + t_{cor1} + t_{cor2} \qquad (6.32)$$

where

t_p = depassivation time

t_{cor1} = time taken to compress the flexible film before exerting pressure on the surrounding brickwork

t_{cor2} = time taken to cause cracking after the flexible film is compressed

= M_{st}/j_{cor} where M_{st} is to be computed from Eq.6.31.

Estimation of V_r

Volume expansion ratio V_r, given by Eq. 6.19, depends on the type of rust compound produced. V_r calculated by Eq. 6.19 for some typical compounds are shown in Table 6.1. Densities of different compounds shown in Table 6.1 are taken from Handbook (133). It should be noted that one atom of Fe produces one molecule of $Fe(OH)_2$,$Fe(OH)_3$ and $FeO(OH)$ whereas three atoms of Fe produce one molecule of Fe_3O_4. The volume expansion ratio is approximately 2 for black rust whereas V_r varies from 3.85 to 4.42 for red rust depending on the extent of hydration as can be seen from Table 6.1. Thus production of red rust is more dangerous than black rust for causing crack to the brickwork. However, the proportions of different compounds formed in actual situation is not known.

Depassivation Time t_p

After the alkaline environment surrounding the reinforcement is lost, the steady state corrosion begins. If the protective layer to the reinforcement is broken repeatedly by some other means (e.g., repeated drying and wet condition) or repeated loads, steady state corrosion begins. Depassivation time t_p depends on the following factors:

Table 6.1 : Volume Expansion Ratio for Different Rust Compounds

Rust Compound	Density ρ_{rust} (gm/cm3)	Mol.weight of compound	$\frac{M_{rust}}{M_{st}}$	$\frac{\rho_{st}}{\rho_{rust}}$	V_r
$Fe(OH)_2$	3.4	89.85	1.609	2.309	3.71
$Fe(OH)_3$	3.4- 3.9*	106.85	1.913	2.013-2.309	3.85-4.42
$FeO(OH)$	4.28	88.85	1.591	1.834	2.918
Fe_3O_4	5.18	231.55	1.382	1.520	2.100

* Depends on the extent of hydration

Note: At. wt. of Fe = 55.85, $\rho_{st} = 7.85$ gm/cm^3

(i) Rate of carbonation of cement mortar which causes acidic environment.

(ii) Atmosphere having high concentration of sulphur dioxide which also destroys the alkaline environment.

(iii) Sulphate content of bricks. Sulphates leached out from brick can destroy the alkaline environment.

(iv) If the reinforcement touches the brick, there is no alkaline environment which will protect the reinforcement from corrosion.

(v) Percent of water absorption of bricks.

Quantitative assessment of depassivation time is very difficult because it depends on numerous factors as given above.

Depassivation time can be increased as follows:

(i) Using bricks of low permeability and low suction rate.

(ii) Using dense mortar which will give low permeability.

(iii) Careful supervision that reinforcements donot touch bricks.

(iv) Increasing clear cover to the reinforcements.

(v) Plastering of all exposed surfaces. Water proof plastering or protective coating in cases where the brickwork is exposed to high humid or moist condition.

Estimation of t_{cor1}

Let c' be the thickness of flexible film around the reinforcement. Assuming all round rusting occurs, the time

taken to compress the flexible film before exerting pressure to the surrounding brickwork can be expressed as

$$t_{cor1} = \frac{c'}{j_r(V_r - 1)} \qquad (6.33)$$

where j_r = average penetration rate per unit time

V_r = volume expansion ratio.

Estimation of thickness of flexible film has considerable influence on the initial time taken (t_{cor1}) before developing pressure on the surrounding brickwork. Estimation of c' would involve a very sophisticated instrumentation. The value of c' at the most can be assumed as the permissible crack width of water retaining structures. It is only an rough estimate. c' = 0.01 mm is assumed in the numerical illustration.

Estimation of t_{cor2}

After the flexible film is compressed, further corrosion of reinforcement will start developing pressure on the surrounding brickwork which in turn will cause cracking. Assuming all round rusting as before,

$$t_{cor2} = \frac{M_{st}}{j_{cor}} = \frac{M_{st}}{j_r \; \pi \; D \; \rho_{st}} \qquad (6.34)$$

where j_r = average penetration rate per unit time and M_{st} is to be calculated from Eq. 6.31. Eqs.6.33 and 6.34 are based

on the assumptions that c' and j_r are both small compared to the diameter (D) of the reinforcement. Application of the above equations is illustrated by the following example.

Young's modulus of masonry (E) is found to be between 400 to 1000 times the masonry crushing strength (134). Hendry (36) gives an approximate relationship

$$E = 700 \, f_w$$

where f_w is the crushing strength of masonry.

Long term Young's modulus can be taken as one half to one third of the instantaneous Young's modulus (34,36). Poisson's ratio of brickwork can vary from 0.11 to 0.20(34). The proportion of brick and mortar enclosed by the assumed cylinder (shown in Fig. 6.2(a)) is important to select the appropriate value of E. If the area of mortar in the assumed cylinder is dominant as compared to the area of brick, it may be reasonable to take Young's modulus and Poisson's ratio of mortar in the numerical calculation. However, these values will of the same order of magnitude to the values of masonry. Let,

$D = 12$ mm, $d_c = 25$ mm, $s = 95$ mm and $f_w = 10$ N/mm^2

If 1:3 mortar is used, average tensile strength of mortar $f_{wt}=1.1$ N/mm^2. Assuming red rust is produced, $V_r=4$.

Let, $E = \dfrac{700 \, f_w}{2} = 3500$ N/mm^2 and $\nu = 0.15$.

Since $\dfrac{s-2D}{2} = 35.5$ mm $> d_c = 25$ mm, inclined cracks are expected from the condition given by Eq. 6.24. Assuming

plane stress condition, Eq. 6.29 gives $a_r = 0.004076$. Therefore, Eq.6.32 gives the amount of steel to be consumed to develop inclined cracks as

$$M_{st} = \frac{\rho_{st}\pi}{(4-1)} (1.1) \left(25+ \frac{12}{2}\right) (0.004076)$$

$$= 0.04633\ \rho_{st}\pi$$

Field exposure studies on concrete cubes having reinforcement at varying cover thickness have shown all round rusting at 13 mm cover thickness (135). The average corrosion rate was found to be 0.002 to 0.006 mm/year at different places for concrete free of salt (135). Let the corrosion rate (j_r) be 0.001 mm/year for the 25 mm cover and t_{cor2} is calculated from Eq. 6.34,

$$t_{cor2} = \frac{0.04633\ \rho_{st}\pi}{0.001(12)\ \rho_{st}\pi} = 3.86 \text{ yrs.}$$

Assuming c' = 0.01 mm, t_{cor1} is calculated by Eq. 6.33,

$$t_{cor1} = \frac{0.01}{0.001\ (4-1)} = 3.33 \text{ yrs.}$$

Carbonation could reach 50 mm in less than 20 years with the types of mortar likely to be used in reinforced brickwork(136). Although there is uncertainity about the rate of penetration, let 15 years be the time to carbonize 25 mm cover thickness. Thus,

$$t_p = 15 \text{ yrs}$$

and time to cracking or spalling of brickwork is given by Eq. 6.32,

$$t_{cr} = 15 + 3.33 + 3.86 = 22.19 \text{ yrs.}$$

Figures used in this example are correct to the order of magnitude but differs from situation to situation. Even if the existance of thin flexible film is ignored, a conservative estimate of t_{cr} would be

$$t_{cr} = 15 + 3.86 = 18.36 \text{ yrs.}$$

It is clear that after the depassivation occurs, the duration of steady state corrosion to cause cracking of brickwork is only 3.86 years which signifies that the depassivation time (t_p) is the controlling factor. Thus, care has to be taken to increase the depassivation time to avoid cracking or spalling of brickwork. However, after the mortar is carbonized, simultaneous presence of water and oxygen is necessary for rusting of reinforcement. Continuous wetting and drying can give more rusting. Some of the measures which could be taken to increase the depassivation time were discussed earlier.

6.4 Reliability Analysis of RBB when Strength Deteriorates with Time

Moment capacity of a RBB section is a random variable since it is a function of other random variables f_w and f_y as seen in Chapter 4. The moment capacity of the beam can

decrease with time if the beam is exposed to corrosive environment. The decrease in moment capacity may be either due to corrosion of reinforcement or due to decrease in strength of masonry or both depending on the type of corrosive environment.

Let $M(t)$, the moment capacity after time t, be a deterministic function of initial (random) moment capacity M_r and expressed as

$$M(t) = M_r\, g(t) \tag{6.35}$$

where M_r = initial moment capacity at $t = 0$ (random variable)

$g(t)$ = a decreasing positive function of time t.

Different forms of deterioration function $g(t)$ are possible depending on the type of deterioration. When the loads are applied either in equal interval or at prescribed instants, the life of a deteriorating member can be measured in terms of number of load applications. Let the moment capacity of the beam decreases with time as given in Eq. 6.35 and independent of the number of load applications. Let $F_{M_e(i)}(x)$ be the probability distribution function of the ith applied external moment $M_e(i)$ at prescribed instants t_i. Let the loads at prescribed instants be independent of each other. Therefore, $[\prod_{i=1}^{n} F_{M_e(i)}(y(t_i))]\; f_{M_r}(y_o)\, dy_o$ is the probability that the

member with initial moment capacity between y_o and y_o+dy_o will survive first n applied loads(58), when $y(t_i)$ is the moment capacity of the same member at time t_i, i.e., after i-1 load application. $f_{M_r}(y_o)$ is the probability density function of M_r. Following Eq. 6.35, $y(t_i)$ is given by

$$y(t_i) = y_o\, g(t_i) \tag{6.36}$$

Summing over all possible values of y_o, the probability of the member, with resisting strength specified by the probability density function $f_{M_r}(y_o)$, to survive a series of n load applications, i.e., the reliability function is given by

$$L_N(n) = \int_o^{\infty}\Big[\prod_{i=1}^{n} F_{M_e(i)}\,(y(t_i))\Big]\, f_{M_r}(y_o)\, dy_o \tag{6.37}$$

The probability of failure $F_N(n)$ is given by

$$F_N(n) = 1 - L_N(n) = \int_o^{\infty}\Big[1-\prod_{i=1}^{n} F_{M_e(i)}(y(t_i))\Big]\, f_{M_r}(y_o) dy_o \tag{6.38}$$

Eq. 6.38 can be written as

$$F_N(n) = \int_o^{\infty}\Big[1 - \prod_{i=1}^{n}(1-\bar{F}_{M_e(i)}\,(y(t_i)))\Big]\; f_{M_r}(y_o)\, dy_o \tag{6.39}$$

where

$$\bar{F}_{M_e(i)}(y(t_i)) = 1 - F_{M_e(i)}\,(y(t_i))$$

Now, $\prod_{i=1}^{n}(1 - \bar{F}_{M_e(i)}\,(y(t_i))$ can be approximated by

$[1-\sum_{i=1}^{n} \bar{F}_{M_e(i)}(y(t_i))]$ and thus Eq. 6.39 can be written as

$$F_N(n) \approx \sum_{i=1}^{n} \int_0^{\infty} \bar{F}_{M_e(i)}\,(y(t_i))\, f_{M_r}(y_o)\, dy_o \qquad (6.40)$$

The approximation is based on the assumption that the major contribution of the integral in Eq. 6.39 comes from those values of y_o for which $\sum_{i=1}^{n} \bar{F}_{Me(i)}(y(t_i)) \ll 1$.

It follows that

$$f_{M(t_i)}\,(y(t_i))\, dy(t_i) = f_{M_r}(y_o)\, dy_o \qquad (6.41)$$

Hence, Eq. 6.40 becomes

$$F_N(n) \approx \sum_{i=1}^{n} \int_0^{\infty} \bar{F}_{M_e(i)}(y(t_i))\;\; f_{M(t_i)}(y(t_i))\, dy(t_i)$$

$$= \sum_{i=1}^{n} p_f(t_i) \qquad (6.42)$$

where $p_f(t_i)$ is the probability of failure of the member resulting from ith application of external moment and is given by

$$p_f(t_i) = \int_0^{\infty} \bar{F}_{M_e(i)}\,(y(t_i))\;\; f_{M(t_i)}(y(t_i))\, dy(t_i) \qquad (6.43)$$

If all the external moments $M_e(i)$, applied in a sequence, belong to one and the same distribution $F_{M_e}(x)$, Eq. 6.43 becomes

$$p_f(t_i) = \int_0^{\infty} [\, 1 - F_{M_e}(y(t_i))]\cdot f_{M}(t_i)\;(y(t_i))\, dy(t_i) \qquad (6.44)$$

Using Eq. 6.41, the above equation can be written as

$$p_f(t_i) = \int_0^\infty [1 - F_{M_e}(y_o g(t_i))] f_{M_r}(y_o)\, dy_o \qquad (6.45)$$

$p_f(t_i)$ is the probability of failure of the member resulting from i th application of external moment and can be expressed in an alternative form as

$$p_f(t_i) = P\,(M(t_i) \leq M_e)$$

$$= P\,(M_r \leq M_e/g(t_i))$$

$$= \int_0^\infty F_{M_r}(M_e/g(t_i))\, f_{M_e}(M_e)\, dM_e \qquad (6.46)$$

where

$f_{M_e}(M_e)$ = probability density function of external moment M_e

$F_{M_r}(x)$ = probability distribution function of moment capacity M_r.

The probability distribution function $F_{M_r}(x)$ is given in Eq. 4.35. When strengths of masonry and steel follow normal distribution, p_{fu} and p_{fo} given in Eq. 4.35 are to be computed from Eqs. 4.40 and 4.42 respectively.

The reliability function becomes

$$L_N(n) \approx 1 - \sum_{i=1}^{n} p_f(t_i) \tag{6.47}$$

The application of the above equations is illustrated through the following example.

Example 6.1

Strength of masonry f_w and strength of steel f_y are distributed as N(8.96, 1.26) N/mm^2 and N(449.15, 22.457) N/mm^2 respectively. The external bending moment on the section in every year is independent and lognormally distributed with mean 7 kNm and coefficient of variation 20 percent. Moment capacity of the RBB section (details of the section are given below) decreases with time to 75 % of its initial strength at the end of 50 years period. Find the probability that the section will fail in 40 year period, when:

(a) moment capacity decreases asymptotically with time

(b) moment capacity decreases parabolcally with time having a zero gradient to start with.

Details of the section are as follows:

$b = 350$ mm , $d = 175$ mm

$A_{st} = 235.62$ mm^2 (3 Nos. 10 Φ bars)

The external moment is given by:

$$M_{em} = 7 \text{ kNm} \quad \text{and} \quad \delta_{M_e} = 0.20$$

Using the mean values of the strengths of the masonry and steel, the mean moment capacity is computed from Eq. 4.11 and it is

$$M_{rm} = 16.413 \text{ kNm.}$$

The parameters of lognormal distribution are obtained from Eqs.4.44 and 4.45 as:

$$\sigma_{ln} = 0.198$$
$$M_{ln} = 6.864 \text{ kNm.}$$

The ratio of the mean values of the moment capacity and external moment at the initial stage is

$$\frac{M_{rm}}{M_{em}} = \frac{16.413}{7} = 2.3447.$$

(a) Asymptotic deterioration

The deterioration function is assumed as

$$g(t) = e^{-at}$$

At t = 50 yrs and for g(t) = 0.75, the value of a works out to be a =0.00575.

Therefore, $g(t) = e^{-0.00575\, t}$

The graph of the above deterioration function is shown by curve A in Fig.6.3. The probability that the section will

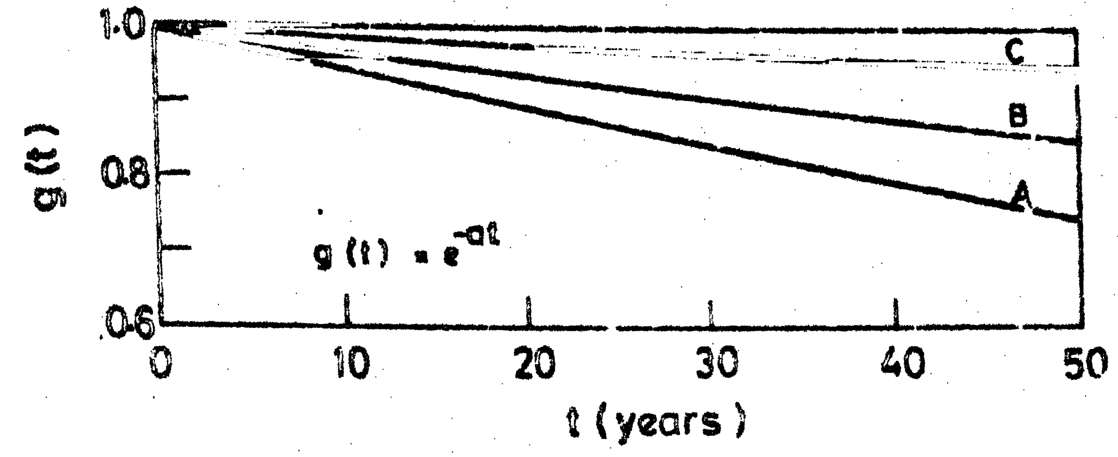


Fig. 6.3 Asymptotic deterioration

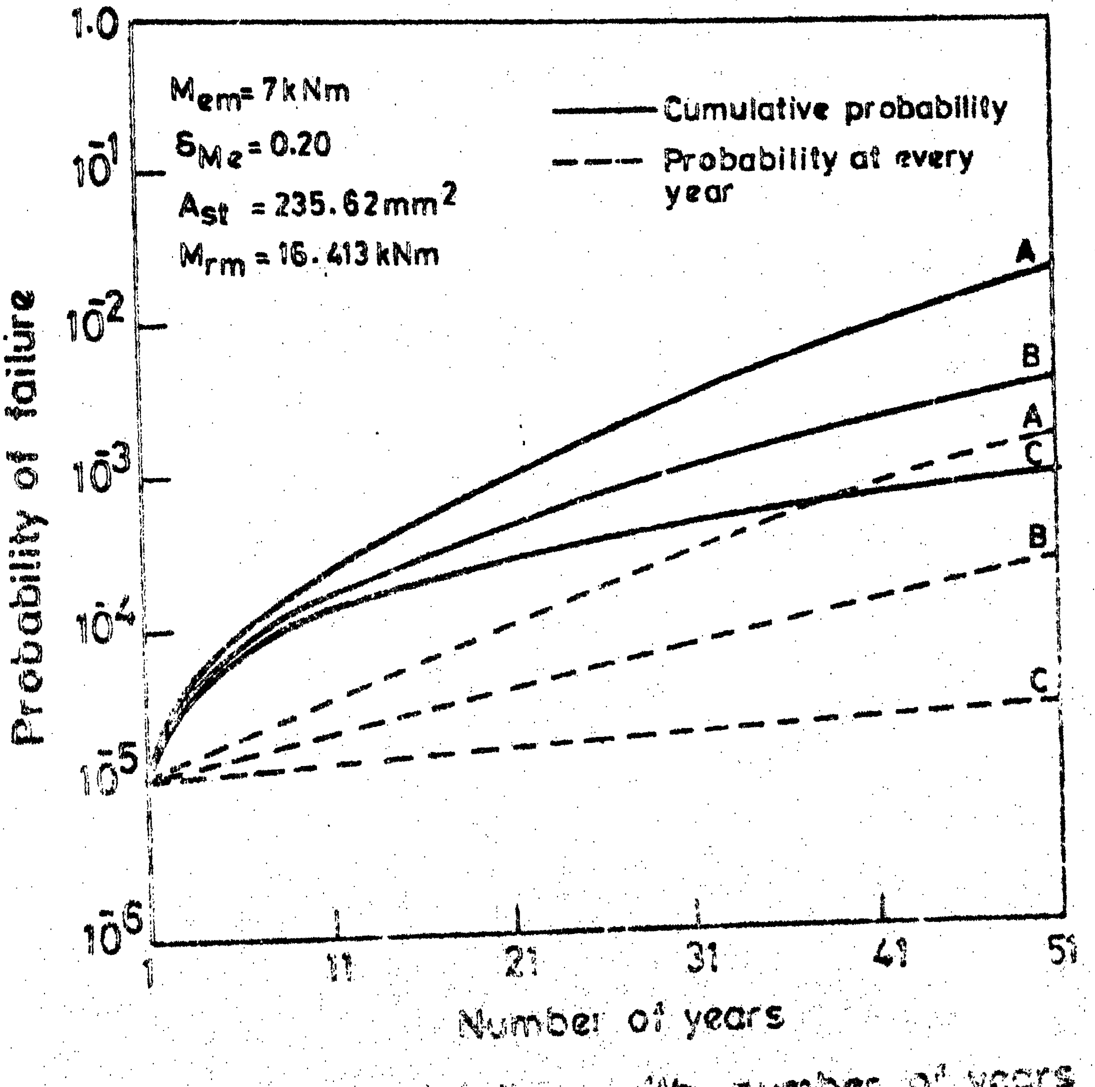


Fig. 6.4 Probability of failure with number of years

fail in 40 year period is computed from Eq.6.42 and it is given by

$$P(N \leq 40) \approx \sum_{i=1}^{40} p_f(t_i) = 7.336 \times 10^{-3}$$

$p_f(t_i)$ in Eq. 6.42 is computed from Eq. 6.46. The probability distribution function $F_{M_r}(x)$ used in Eq. 6.44 is given in Chapter 4. The probability of failure at the end of first year is found to be 1.195×10^{-5}. It can be noticed that the probability of failure in forty year period has increased by 614 times that of the first year's probability of failure. The probability of failure of the section in forty year period in case the strength of the section remains constant with time (i.e., no deterioration) will be about forty times that of the probability of failure in the first year. The twenty five percent deterioration has increased the probability of failure by many times.

Curves A, B and C in Fig. 6.3 represent 25%, 15 % and 5 % decrease in strength in 50 years respectively. The probability of failure at every year and the cumulative probability of failure with number of years corresponding to these degrees of deteriorations (shown in Fig. 6.3) are shown in Fig. 6.4. When the degree of deterioration is increased from 5% to 15%, the cumulative probability of failure at 51st year increased from 9.318×10^{-4} to 4.008×10^{-3}, i.e., 3 times increase in degree of deterioration has increased

the probability of failure by 4.3 times. The cumulative probability at 51st year is 2.156×10^{-2} (i.e. 1804 times increase with respect to the 1st year's probability of failure) when the degree of deterioration is 25 % as can be seen from Fig. 6.4. As the degree of deterioration is increased by 5 times (i.e., 5 % to 25 %), the cumulative parabability of failure has increased by 23 times.

(b) Parabolic deterioration

Assume the deterioration function to be

$$g(t) = a_1 + a_2 t + a_3 t^2$$

At $t = 0$, $g(0) = 1$ and $\frac{d}{dt} g(t) \Big|_{at\ t=0} = 0$

give $a_1 = 1$ and $a_2 = 0$. Hence, the deterioration function becomes

$$g(t) = 1 + a_3 t^2$$

At $t = 50$ yrs and for $g(t) = 0.75$, the value of a_3 works out to be $a_3 = -0.0001$.

Therefore, $g(t) = 1 - 0.0001\ t^2$

This type of deterioration can be expressed in a general form as

$$g(t) = 1 - at^2$$

The graph of the above deterioration function is shown by curve A (25 % decrease) in Fig. 6.5. The probability that

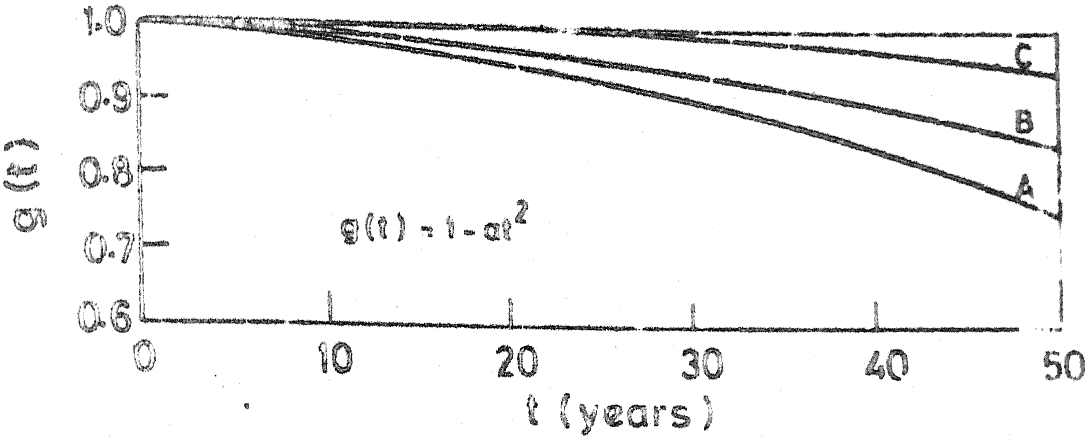


Fig. 6.5 Parabolic deterioration

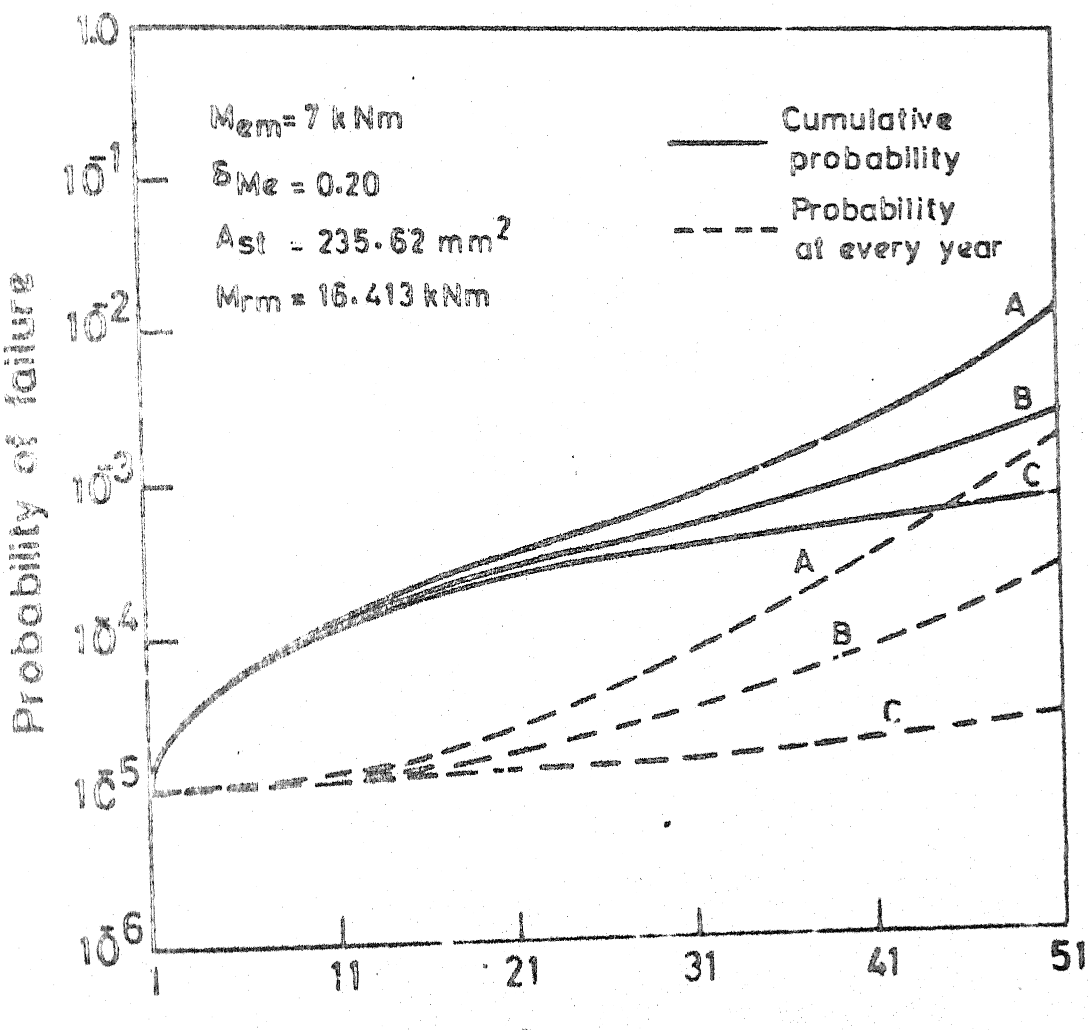


Fig. 6.6 Probability of failure with number of years

the section will fail in 40 years period is computed from Eq. 6.42 and it is given by

$$P(N \leq 40) \approx \sum_{i=1}^{40} p_f(t_i) = 2.233\text{x}10^{-3}$$

The probability of failure at the end of first year is found to be $1.058\text{x}10^{-5}$. The probability of failure in 40 years period has increased by 211 times that of the first year's probability of failure.

Curves A, B and C in Fig. 6.5 represent 25%, 15% and 5 % decrease in strength in 50 years respectively. The probability of failure at every year and the cumulative probability of failure with number of years corresponding to these degrees of deteriorations (shown in Fig.6.5) are shown in Fig. 6.6. When the degree of deterioration is increased from 5 % to 15 %, the cumulative probability of failure at 51st year increased from $8.022\text{x}10^{-4}$ to $2.671\text{x}10^{-3}$, i.e., 3 times increase in the degree of deterioration has increased the probability of failure by 3.3 times. The cumulative probability of failure at 51st year is $1.287\text{x}10^{-2}$ (i.e., 1216 times increase with respect to the 1st year's probability of failure) when the degree of deterioration is 25 %. As the degree of deterioration is increased by 5 times (i.e., 5 % to 25 %), the probability of failure has increased by 16 times. It is to be noted that in case of asymptotic

deterioration, the probability of failure increases by 23 times whereas for parabolic deterioration it is 16 times. Thus, the type of deterioration function and the degree of deterioration have a direct impact on the cumulative probability of failure.

6.5 Discussions and Conclusions

The alkaline environment caused by the hydration of cement mortar gives a natural protection to the reinforcement. Steel remains protected and immune to corrosion as long as the pH value of the surrounding medium is around 10 to 13. Due to action of salts present in the brick or due to carbonation, the pH value of mortar decreases and consequently the danger of corrosion becomes inevitable. The volume of rust is two to four times that of the corresponding amount of steel reacted depending on the type of rust compounds formed. The increased volume of the material due to rusting exerts pressure on the surrounding brickwork causing cracking and spalling of brickwork. Spalling of mortar and splitting of bricks were observed in some cases (119). Estimation of time of cracking or spalling of brickwork due to corrosion of reinforcement is formulated. The time of cracking depends on depassivation time, the diameter and spacing of reinforcement, Poisson's ratio and Young's modulus of brickwork, average tensile strength of brickwork, type of rust compound produced

and the rate of corrosion. Simultaneous presence of water and oxygen is necessary for corrosion of reinforcement after the mortar is carbonized. Once the steady state corrosion starts after the depassivation period, the time required for cracking or spalling of brickwork is estimated to be about three to four years for a corrosion rate of 0.001 mm/yr. The depassivation time which is the period before steady state corrosion starts, is the dominating factor that governs the total time elapsed before cracking or spalling of brickwork takes place. Thus, care has to be taken to control the depassivation time by the measures discussed earlier to increase the durability of reinforced brickwork.

Due to corrosion, the effective area of reinforcement decreases which results in decrease of moment capacity. Deterioration in bond between reinforcement and brickwork due to rusting is not yet understood. The moment capacity of a RBB section can also decrease with time due to decrease in brickwork strength by chemical reaction. Moment capacity of the section at any time is assumed to be a decreasing positive function of the initial (random) moment capacity but independent of the load applications. The reliability analysis considering the decrease of moment capacity with time is formulated when there are series of independent load applications. The probability of failure can be estimated by

summing the probabilities of failure at each load application considering the deteriorated strength of the section. Two types of deterioration models namely asymptotic and parabolic deteriorations are considered in the illustrative example. However, the actual type of deterioration will depend on the type of exposure condition. Cumulative probability at 51st year is used for comparison purpose. For 25% degree of deterioration, the cumulative probability of failure has increased by 1804 times and 1216 times with respect to first year's probability of failure for asymptotic and parabolic deteriorations respectively. For a 5 times increase in degree of deterioration (from 5 % to 25 %), cumulative probabilities of failure are found to increase by 23 times and 16 times for asymptotic and parabolic deteriorations respectively. Thus, the type of deterioration function and the degree of deterioration have direct impact on the cumulative probability of failure.

The probability of failure at each year with different M_{rm}/M_{em} ratio is plotted in Fig. 6.7 for both asymptotic and parabolic deteriorations. It can be seen from Fig. 6.7 that the probability at every year depends on the type of deterioration model and decreases with M_{rm}/M_{em} ratio. The cumulative probability of failure with number of years for different M_{rm}/M_{em} ratio is shown in Fig. 6.8

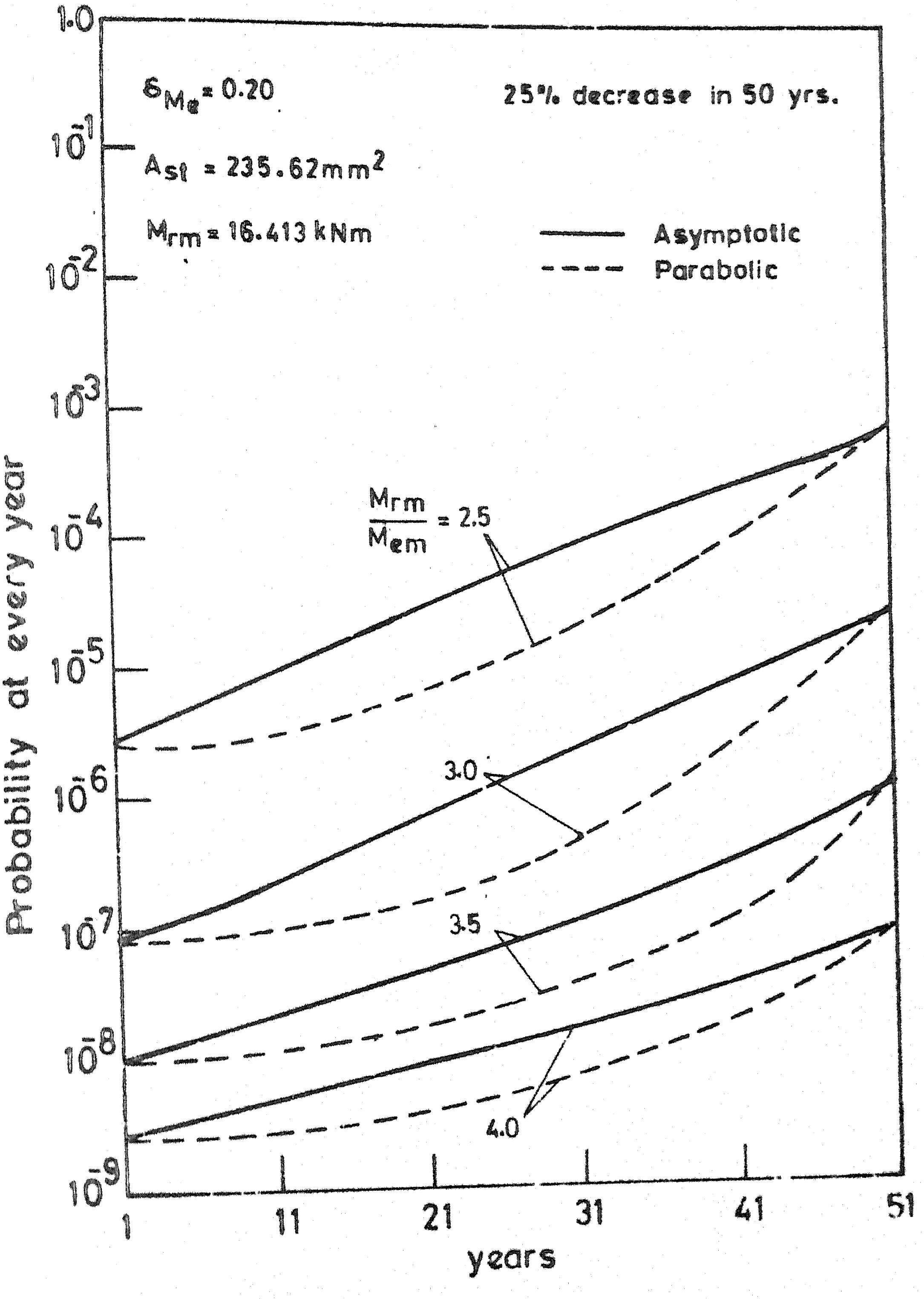


Fig. 6.7 Probability of failure at different years

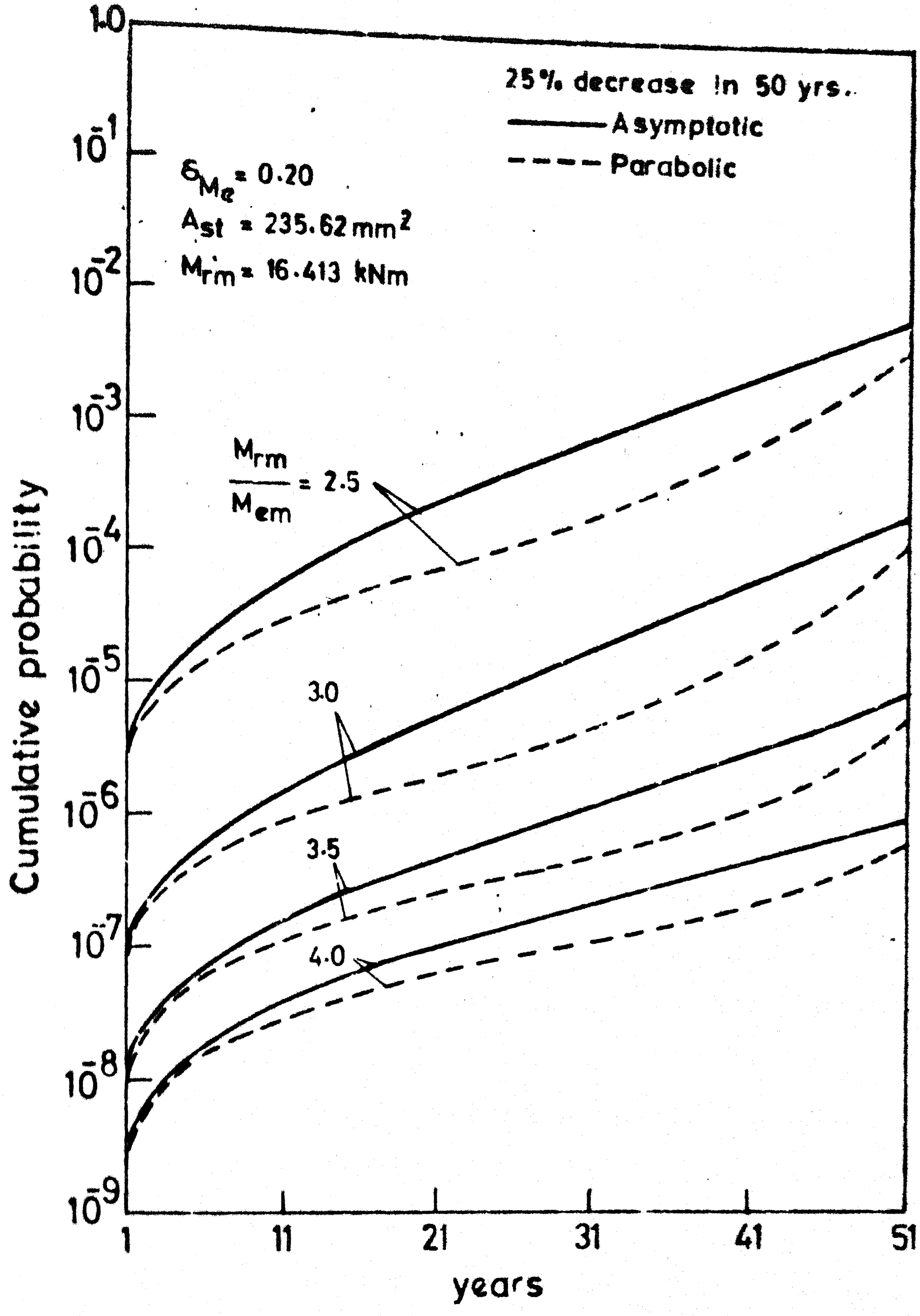


Fig. 6.8 Cumulative probability of failure with number of years

for both types of deteriorations. As M_{rm}/M_{em} ratio increases the probability of survival also increases as expected. M_{rm}/M_{em} ratio is the initial load factor applied to the mean values. For $M_{rm}/M_{em} = 2.5$, the cumulative probabilities of failure at 51st year are 7.288×10^{-3} and 4.425×10^{-3} for asymptotic and parabolic deteriorations respectively. Similarly, for $M_{rm}/M_{em} = 3.0$, the cumulative probabilities of failure are 2.358×10^{-4} and 1.415×10^{-4} for asymptotic and parabolic deteriorations respectively. Hence, an increase in M_{rm}/M_{em} ratio by 1.2 times has decreased the probabilities of failure by 30.6 and 31.3 times for asymptotic and parabolic deteriorations respectively. A small increase in M_{rm}/M_{em} ratio has decreased the cumulative probability of failure considerably. If M_{rm}/M_{em} ratio is increased by 1.4 times (i.e., from 2.5 to 3.5), the probabilities of failure reduces from 7.288×10^{-3} to 1.017×10^{-5} and from 4.425×10^{-3} to 6.404×10^{-6} for asymptotic and parabolic deteriorations respectively as can be seen from Fig. 6.8. Thus, selection of initial load factor (i.e., M_{rm}/M_{em} ratio) has got a direct impact on the probability of failure. The probability of failure is also sensitive to the type and degree of deterioration. In a design problem, the probability of failure for a given design life has to be checked. For a preassigned reliability; the initial load factor has to be increased depending on the type and degree of deterioration.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 General

Statistical analysis of different properties of brick, mortar and masonry is carried out and presented in Chapter 2. Chi-square test is conducted on different random variables to fit suitable probability distributions. Reliability of chi-square test is discussed and presented in Chapter 3. Reliability analysis and reliability based design procedure incorporating statistical variations in various parameters are presented in Chapters 4 and 5 respectively. Chapter 6 presents a study on the durability aspect of reinforced brickwork, in particular, corrosion of reinforcement. The conclusions and recommendations based on the work carried out in this thesis are summarized in the present chapter. Suggestions for further research are indicated at the end.

7.2 Conclusions and Recommendations

Nine different brick samples each containing about hundred bricks were studied for statistical variations in their properties. The first eight samples were taken from eight different manufacturers at random while the ninth contains several sets supplied by different building supervisors for quality analysis. The ninth sample can be

considered completely random. Seven different properties namely length, breadth, height, dry density, wet density, percentage of water absorption and flatwise compressive strength of each brick were measured and statistical analysis was carried out. Chi-square test was applied to test whether the data fits a particular probability distribution.

The coefficient of variation in length and breadth of brick was found to lie within 2.8 percent indicating a high consistency in the dimensional accuracy of the hand made bricks. The coefficient of variation in length of brick from a single manufacturer was within 2 percent and that associated with many manufacturers was within 2.1 percent. The variation in height of brick appears to be more than that of length and breadth. Even in this case, the coefficient of variation was limited to 3.4 percent. Dimensions of bricks manufactured from a single manufacturer showed a very small variability with a tendency towards deterministic value. The dimensions can be assumed to be normally distributed for practical purposes.

Density of dry bricks of different individual manufacturers followed normal distribution. The mean values of dry density of bricks of different samples differed reasonably but the coefficient of variation of each sample was found to be within 5 percent. This again

reflects the dimensional accuracy of the hand moulded bricks and also the compaction and homogeneity in preparing the soil for brick making. The mean value of the percentage water absorption of brick samples varied from 13.41 to 18.47 with the coefficient of variation ranging from 4.7 to 17.7 percent. Normal and lognormal distributions did not fit the water absorption data in five samples but in other three samples normal distribution can be accepted at 5 percent significance level. In these samples, normal distribution appears to be a better model than lognormal distribution. Normal distribution can be accepted to represent the variability of percentage water absorption for practical purposes.

Compressive strength of brick followed normal distribution at 2.5 percent significance level. Within the first eight samples, first three samples were collected from different manufacturers of Kanpur zone and the rest five were collected from Faroke in Kerala State. Mean compressive strength of brick samples from Kanpur zone varied from 14.57 N/mm^2 to 23.69 N/mm^2 with a coefficient of variation ranging from 22.5 to 24.6 percent whereas mean compressive strength of Faroke zone varied from 7.6 N/mm^2 to 14.16 N/mm^2 with a coefficient of variation ranging from 15.1 to 18.6 percent. This shows that bricks of Kanpur zone are much stronger than those made in Faroke. Normal

distribution did not fit the random lot (the sample which contains several sets supplied by different building supervisors) at 1 percent significance level whereas lognormal distribution fits at 18 percent significance level.

Cement-sand mortar cubes of three different mixes (1:3, 1:4 and 1:5) were cast in the laboratory and tested after 28 days, to study the variability of mortar strength. The coefficient of variation of the strength of mortar cubes in three different mixes varied from 10 to 18 percent. The coefficient of variation of strength of field cubes may be expected to be about 20 percent more than that observed in the laboratory, and could be anywhere between 12 to 22 percent. This is comparable with the quality of M15 concrete produced from nominal mix. Compressive strength of mortar (laboratory specimens) of each mixes was found to follow normal distribution at 10 percent significance level. Field data should be collected for a proper understanding of the variability of mortar strength in actual practice.

Thickness of mortar joint was measured to study the variability in joint thickness. Three different buildings of exposed brick walls were selected for study. Both bed joint and vertical joint thicknesses were measured at different locations of the wall selected at random. The

coefficient of variation of bed joint thickness varied between 12 to 14 percent whereas that of vertical joint thickness varied from 13 to 20 percent. The lowest and highest measured thicknesses were observed to lie within ± 5 mm of the mean which signifies a very good quality control at the time of brick laying. Thickness of mortar joint followed normal distribution at 1 percent significance level.

The strength of brick masonry depends on the quality of brick, mortar, joint thickness and joint layout. Due to non availability of field data, the strength of masonry was generated by Monte Carlo simulation on a digital computer from the randomly generated data of brick and mortar strength using their individual statistical properties. Strength of brick and mortar were taken as normally distributed. Two deterministic formulae relating strength of brick, mortar and masonry were used for simulation, and given by

$$f_w = k_w \sqrt{(f_b\ f_m)} \qquad (7.1)$$

$$f_w = A(0.758 + 0.155\ f_b + 0.0082\ f_b f_m) \qquad (7.2)$$

where f_b, f_m and f_w are strengths of brick, mortar and masonry respectively. Eq. 7.2 was derived from the formula given in SCPI recommendation (13) whereas Eq. 7.1 is based on limited experimental investigation (2).

Simulated data based on Eq. 7.1 did not follow normal distribution even at 0.1 percent significance level. The central region of the simulated data matched with normal distribution but mismatching was observed at either tail. Simulated data based on Eq. 7.2 followed normal distribution at 0.5 percent significance level. Strength of masonry may still be assumed as normally distributed random variable.

Chi-square test and Kolmogorov - Smirnov test (K-S test) are the two popular tests used to examine whether a data of a particular random variable follows an assumed probability distribution. Usual chi-square test with equal class interval for testing the suitability of a hypothesized probability distribution depends on three arbitrary decisions such as the choice of the starting point, number of classes and width of classes. Hence this test can lead to doubtful acceptance or rejection of the null hypothesis. The three arbitrary choices in usual chi-square test can be made to a single variable if the test is conducted with classes of equal probability. Even if the classes of equal probability is used, the computed value of chi-square statistic is observed to change randomly with the choice of number of classes (k) and consequently the maximum significance level(p-level) also changes. Hence for a dependable decision, a series of chi-square tests with different number of classes is recommended.

Based on the different p-levels obtained by the series of tests, characteristic p-level may be computed by Eq. 3.15 and a decision can be taken for acceptance of the null hypothesis.

Selection of a particular probability distribution requires engineering or scientific judgement considering the actual physical phenomena. It is possible that two or more probability distributions may appear equally competent models for a random variable considering the actual phenomena. To test which one of the probability distributions is a better model to represent the random variable, the characteristic p-levels for the competent distributions can be compared. The distribution which gives a higher characteristic p-level may be accepted as a better model as compared to the other distributions.

Kolmogorov-Smirnov test (K-S test) does not require grouping of the data into classes as in chi-square test and hence arbitrary choice of number of classes etc. are not involved. When the parameters of the null distribution are completely specified, K-S statistic is independent of the null distribution. But the K-S statistic becomes dependent on the choice of the null distribution when the parameters are not specified and have to be estimated from the data. In this case, standard tables available for use with K-S

test cannot be used. Tables showing the critical values of K-S statistic for different level of significance are presented for uniform, normal and exponential distributions.

Ultimate moment capacity of a reinforced brick beam (RBB) section is a function of geometric proportions of the section, area of reinforcement, and strengths of the masonry and reinforcement. The section can fail as an under-reinforced or as an over-reinforced. Due to random variations in the strengths of masonry and steel, there exists a probability that a RBB section will fail as an over-reinforced even though the section is designed as an under-reinforced based on deterministic analysis. Probability of failure (p_f) of a RBB section is the sum of (i) the probability of failure of the section as an under-reinforced (p_{fu}) and (ii) the probability of failure of the section as an over-reinforced (p_{fo}).

The computation of failure probabilities p_{fu} and p_{fo} involves evaluation of multiple integrals and depends on the probability distribution of strengths of masonry and steel. The probabilities p_{fu} and p_{fo} are found to be sensitive to the relative magnitudes of the coefficients of variation of strengths of masonry and steel. The probability of failure of the section increases with the increase in coefficients of variation of strengths of masonry

and steel for a given external moment. If the area of steel in a RBB section is increased, the probability of failure as an under-reinforced (p_{fu}) decreases and the probability of failure as an over-reinforced (p_{fo}) increases, however the total probability of failure (p_f) decreases. It is observed that p_f decreases to a limiting value equal to p_{fo} as the area of steel is increased to a very large value, which signifies that the probability of failing as an under-reinforced is almost equal to zero for heavily over-reinforced sections. For probabilistic external moment, the probability of failure increases as the coefficient of variation of external moment increases.

A large number of values of moment capacity (M_r) are generated by Monte Carlo simulation on a digital computer from randomly generated data of strengths of masonry and steel conforming to their individual statistical properties. Histograms of the simulated samples showed a marked negative skewness. As the reinforcement ratio (A_{st}/bd) is increased towards the balanced section value, the distribution of M_r becomes more and more skewed to the right. For a highly under-reinforced section (deterministically), moment capacity M_r is found to be normally distributed.

The design of a RBB section involves calculation of steel area and geometrical proportion such that the section

can resist the external design bending moment. In a probabilistic design, area of steel and geometry of the section are so proportioned that the probability of moment capacity being less than external bending moment is within a preassigned probability of failure. A method which involves solution of an integral equation to find the area of steel or geometrical proportions of the section for a prescribed propability of failure or reliability is presented in Chapter 5. It is found that the coefficients of variation in strengths of masonry and steel have direct impact on the design.

Limit state design method, based on semi-probabilistic approach, incorporates statistical variations in some of the design variables through partial safety factors applied to loads and strengths of materials. Partial safety factors, required for a target reliability, are found to be dependent on the relative magnitudes of the coefficients of variation associated with loads and strengths of materials. For a practical range of coefficients of variation (as discussed in Chapter 5) and for an accepted probability of failure of 10^{-5}, the partial safety factors applied to the mean values of dead and live loads are found to be 1.3 and 1.8 respectively. Corresponding partial safety factor for strength of masonry is found to be 1.75 to 2.5. Partial

safety factor of 1.15 is used for strength of steel. Original draft code BS 5628: Part 2.(48) recommends a value of 2.5 to 2.8 for the partial safety factor applied to strength of masonry at limit state of strength in bending. The revised draft code (BS 5628: Part 2) has now suggested a value of 2 to 2.3. A partial safety factor of 2 to 2.5 for strength of masonry is recommended for limit state design of RB beam section in flexure, when partial safety factors 1.3 and 1.8 are used for dead and live loads (to be applied to the mean values) respectively.

Corrosion of reinforcement in reinforced brickwork construction appears to be a serious problem from serviceability point of view. Rusting of reinforcement increases the volume and causes thrust to the surrounding brickwork resulting in cracking and spalling of brickwork. The volume of rust is two to four times of the corresponding volume of steel reacted depending on the type of rust compounds formed. Estimation of time of cracking or spalling of brickwork due to corrosion of reinforcement is formulated. The cracking of brickwork depends on depassivation time, the diameter and spacing of reinforcement, cover to the reinforcement, Poisson's ratio and Young's modulus of brickwork, average tensile strength of brickwork, type of rust compounds produced and rate of corrosion. The depassivation time which

is the period before steady state corrosion starts, is the dominating factor that governs the total time elapsed before cracking or spalling of brickwork. Thus, care has to be taken to control the depassivation time.

Due to corrosion, the effective area of reinforcement decreases which results into decrease in the moment capacity. The moment capacity of a RBB section can also decrease with time due to decrease in brickwork strength by the action of aggressive environment. Reliability analysis considering the decrease in moment capacity with time is formulated and presented in Chapter 6. Two types of deterioration models namely asymptotic and parabolic deteriorations are considered. However, the actual type of deterioration will depend on the type of exposure. As the moment capacity of a RB section decreases with time, the probability of failure at every year due to external moment increases and as a consequence the cumulative probability of failure increases. For 25 percent decrease in moment capacity in 50 yrs, the cumulative probability of failure at 51st year is found to increase by 1804 times that of the first year's probability of failure for asymptotic deterioration. For parabolic deterioration the cumulative probability of failure increases by 1216 times that of the first year's probability of failure. It is found that the type of deterioration and the degree of

deterioration have a direct impact on the cumulative probability of failure. Cumulative probability of failure at a particular year depends on the initial load factor chosen. An increase of load factor from 2.5 to 3.0 has reduced the cumulative probability of failure at 51st year from 7.288×10^{-3} to 2.358×10^{-4} for asymptotic deterioration. Similar findings are observed for parabolic deterioration. Selection of initial load factor has got a direct impact on the probability of failure. In a design problem, the initial load factor has to be increased depending on the type and degree of deterioration to achieve a preassigned reliability.

7.3. Suggestions for further Research

Dimensional variability of RBB section was not taken into consideration for reliability analysis. Investigation of dimensional variability in actual practice is suggested. Effect of variability of joint thickness on the strength of RBB can also be investigated. Ultimate strain and Young's modulus of masonry can be taken as random variable in the reliability analysis and data may be collected to establish mutual dependence of the above variables.

Extensive data need to be collected on the strengths of mortar and masonry from field, and the decrease in strength

with time in different exposure conditions. Asymptotic and parabolic types of deterioration models are considered in the present work. It is possible that the actual deterioration data may fit into one of these models. Collection of this type of data requires considerable time. National Organisations can take up a long term programme for collection of data on the deterioration under different exposure conditions.

Research may be directed on the quantitative estimation of strength deterioration with time under different exposure conditions. Effect of corrosion of reinforcement on the bond between reinforcement and masonry may be investigated to study the exact mechanism of deterioration of moment capacity of RB beams. Strength deterioration due to repeated loading is not considered in the present work. Reliability analysis and design of RBB under repeated load conditions when strength deteriorates with each load application along with deterioration with time due to aggressive environment could be an important direction of further research.

REFERENCES

1. Srivastava, S.N.P., 'Behaviour of Reinforced Brick Beams under Static and Pulsating Loads', M.Tech. thesis submitted at IIT Kanpur, India, July 1977.

2. Dayaratnam, P., Ranganathan, R., Mukhopadhyay, S., and Dasgupta, N., 'Experimental Investigation on Behaviour of Brick and Reinforced Brickwork', Report No. DST/427/3, IIT Kanpur, India, Feb. 1981.

3. Foster, D., 'Reinforced and Prestressed Brickwork-A Review', International Seminar/Workshop on Planning, Design, Construction of Load Bearing Brick Buildings for Developing Countries', New Delhi, Dec. 1981, pp. 229-282.

4. Beamish, R., 'Memoirs of the Life of Sir M.I. Brunel', London, 1862.

5. Plummer, H.C., and Blume, J.A., 'Reinforced Brick Masonry and Lateral Force Design', Structural Clay Products Institute, Washington D .C., 1953.

6. Brebner, A., 'Results on Tests on Reinforced Brick Masonry Structural Members', Technical Report No.38, Public Works Department, Government of India,1923.

7. Parsons, D.E., Stang, A.H., and McBurney, J.W.,'Reinforced Brick Masonry Structures', U.S. Department of Commerce, Bureau of Standards, Journal of Research, Research Paper No. 504, Vol.9, 1932, pp. 749-768.

8. Withey, M.C., 'Tests on Brick Masonry Beams', American Society of Testing of Materials, Proceedings of Thirty-sixth Annual Meeting, Vol.33, Part II, 1933, pp.651-669.

9. CP111: Part 2, 1970, 'British Standard Code of Practice, Structural Recommendations for Loadbearing Walls, Part 2, Metric Units', BSI.

10. IS 1905-1980, 'Indian Standard Code of Practice for Structural Safety of Buildings: Masonry Walls', ISI, New Delhi.

11. ACI Committee 531, 'Building Code Requirements for Concrete Masonry Structures (ACI 531-79)',ACI,Detroit,1979.

12. 'Building Code Requirements for Reinforced Brick Masonry', Hand Book 74, U.S. Department of Commerce, NBS., UAA, October 1960.

13. Gross, J.G., Dikkers, R.D., and Corogan, J.C.,'Recommended Pratice for Engineered Brick Masonry', Structural Clay Products Institute, November 1969.

14. Cutler, J.F., Plewes, W.G., and Mikluchin, P.T.,'The Development of Canadian Building Code for Masonry', SIBMAC Proc., BCRA, 1971, pp.233-239.

15. Gross, J.G., and Dikkers, R.D., 'Building Code Requirements Relating to Load Bearing Brick Masonry', Designing, Engineering and Constructing with Masonry Products, ed. F.B.Johnson, Gulf Publishing Co.,1969,pp.357-365.

16. Haller, P., 'Load Capacity of Brick Masonry', Designing, Engineering and Constructing with Masonry Products, ed. F.B. Johnson, Gulf Publishing Co.,1969,pp.129-149.

17. Suter, G.T., and Hendry, A.W.,'Shear Strength of Reinforced Brickwork Beams: Influence of Shear Arm Ratio and Amount of Tensile Steel', The Structural Engineer, Vol.53,No.6, June 1975, pp. 249-253.

18. Suter, G .T., and Hendry, A.W., 'Limit State Shear Design of Reinforced Brickwork Beams', Proc. B.Ceram.Soc.,1975, pp. 191-196.

19. McBurney, J.W., 'The Effect of Strength of Brick on Compressive Strength of Brick Masonry', Proceedings ASTM, Vol.28,Part II,1928, pp.605-634.

20. Hilsdorf, H.K.,'Investigation into the Failure Mechanism of Brick Masonry Loaded in Axial Compression',Designing, Engineering and Constructing with Masonry Products, ed. F.B. Johnson, Gulf Publishing Co.,1969, pp.34-40.

21. Stafford Smith B., 'The Diagonal Tensile Strength of Brickwork', The Structural Engineer, Vol.48, No.6, June 1970, pp. 219-225.

22. Talbot and Abrams, 'Compressive Strength of Terracotta Block Columns',Bulletin No.27, Univ. of Illinois.

23. Parsons, D.E., 'Factors Affecting the Strength of Masonry of Hollow Units', NBS, Research Paper No.310, 1931.

24. Hoath, S.B.A., Lee, H.N., and Renton, K.H., 'The Effect of Mortars on the Strength of Brickwork Cubes', SIBMAC Proc., BCRA, 1971, pp. 113-116.

25. Francis, A.J., Horman, C.B., and Jeremes, L.E., 'The Effect of Joint Thickness and Other Factors on the Compressive Strength of Brickwork', SIBMAC Proc., BCRA, 1971, pp. 31-37.

26. Carter, C., and Stafford Smith B., 'Structural Behaviour of Masonry Infilled Frames Subjected to Racking Loads', Designing, Engineering and Constructing with Masonry Products, ed. F .B. Johnson, Gulf Publishing Co., 1969, pp. 226-233.

27. Stafford Smith B., and Carter, C., 'Distribution of Stresses in Masonry Walls Subjected to Vertical Loading', SIBMAC Proc., BCRA, 1971, pp. 119-124.

28. Riddington, J.R., and Stafford Smith B., 'Analysis of Infilled Frames Subject to Racking with Design Recommendations', The Structural Engineer, Vol.55, No.6, June 1977, pp. 263-268.

29. Stafford Smith B., 'Composite Action of Masonry Walls with Beams and Frames', Int. Seminar/Workshop on Planning, Design, Construction of Load Bearing Brick Buildings for Developing Countries, New Delhi, December 1981, pp. 189-203.

30. Turnsek, V., and Cacovic, F., 'Some Experimental Results on the Strength of Brick Masonry Walls', SIBMAC Proc., BCRA, 1971, pp. 149-156.

31. Howard, J.W., Hockaday, R.B., and Soderstrum, Wm.K., 'Effect of Manufacturing and Construction Variables on Durability and Compressive Strength of Brick Masonry', Designing, Engineering and Constructing with Masonry Products', ed. F.B. Johnson, Gulf Publishing Co., 1969, pp. 310-316.

32. Grimm, C.T., and Halsell, R.D., 'Quality Control of Brick Masonry Construction in the USA', SIBMAC Proc., BCRA, 1971, pp. 337-342.

33. Thomas, K., 'Structural Brickwork Materials and Performance', The Structural Engineer, Vol.49, No.10, October 1971, pp. 441-450.

34. Grimm, C.T., 'Strength and Related Properties of Brick Masonry', J. of Struct. Div.,Proc. ASCE, Vol.101,ST1, Jan.1975, pp. 217-232.

35. Plummer, H.C., 'Brick and Tile Engineering', Brick Institute of America, Virginia, June 1977.

36. Hendry, A.W.,'Materials and Workmanship', Int. Seminar/ Workshop on Planning, Designing, Construction of Load Bearing Brick Buildings for Developing Countries,New Delhi, Dec. 1981, pp. 31-53.

37. Macchi, G.,'Safety Consideration for a Limit State Design', SIBMAC Proc., BCRA, 1971, pp. 229-232.

38. Hendry,A.W., 'The Lateral Strength of Unreinforced Brickwork', The Structural Engineer, Vol.51, No.2,Feb.1973, pp. 43-50.

39. CP110: Part 1, November 1972, 'Code of Practice for the Structural Use of Concrete, Part 1, BSI.

40. ACI Committee 318, 'Building Code Requirements for Reinforced Concrete (ACI 318-77)', ACI, Detroit,1977.

41. IS 456-1978, 'Indian Standard Code of Practice for Plain and Reinforced Concrete', ISI, New Delhi.

42. Pume,D., 'Design Methods for Brick Masonry Structures in Czechoslovakia', SIBMAC Proc.,BCRA,1971,pp.303-306.

43. BS 5628: Part 1, 1978, 'British Standard Code of Practice for the Structural Use of Masonry, Part 1, Unreinforced Masonry',(with Ammendment upto 30th September 1980),BSI.

44. 'Design Guide for Reinforced and Prestressed Clay Brickwork', Special Publication 91, British Ceramic Research Association, Stoke on Trent, Dec.1977.

45. Jain, S.C., et al., 'Ultimate Flexural and Shear Strength of Reinforced Brickwork', J. of Inst. of Engineers(India), C.E.Division, Pt CI1, Vol.61, July 1980,pp.37-41.

46. Board, R., 'A Theoretical Analysis of Reinforced Brickwork in Bending', British Ceramic Society,Symposium on Load Bearing Brickwork, London, November 1980.

47. Sinha, B.P., 'An Ultimate Load Analysis of Reinforced Brickwork Flexural Members', Int. Journal of Masonry Construction, Vol.1, No.4, 1981, pp.151-155.

48. BS 5628: Part 2,1981 (Draft Code), 'British Standard Code of Practice for Structural Use of Masonry, Part 2, Reinforced and Prestressed Masonry', BSI.

49. Beard, R., 'A View on the Shape of the Stress Block and Material Safety Factors', Symposium on Reinforced and Prestressed Masonry', Inst. of Structural Engineers, London, July 1981, pp. 46-52.

50. Hendry, A.W., 'The Shear Strength of Reinforced Brickwork', Symposium on Reinforced and Prestressed Masonry', Inst. of Structural Engineers, London, July 1981 ,pp.29-37.

51. Freudenthal,A.M., 'Safety of Structures', Transactions ASCE, Vol.112, 1947, pp. 125-159.

52. Asplund, S.O., 'Probabilities of Traffic Loads on Bridges', Proc. ASCE, Vol.81, No.585, 1955.

53. Pugsley, A.G., 'Structural Safety', J.of the Royal Aeronautical Society, Vol.59, June 1955, pp.415-421.

54. Freudenthal, A.M., 'Safety and the Probability of Structural Failure', Transactions ASCE, Vol.121,1956, pp. 1337-1375.

55. Julian, O.G., 'Synopsis of First Progress Report of Committee on Factors of Safety', J. of Struct. Div., Proc. ASCE, Vol.83, ST-4, July 1957, pp.1316-1322.

56. Brown, C.B., 'Concepts of Structural Safety', J.of Struct. Div., Proc. ASCE, Vol.86, ST-12, Dec. 1960, pp.39-57.

57. Freudenthal, A.M., 'Safety, Reliability and Structural Design', Transaction ASCE, Vol.127, Part II, Proc.Paper 3372, 1962, pp.304-319.

58. Freudenthal, A.M., Garrelts, J.M., and Shinozuka, M., 'The Analysis of Structural Safety', J.of Struct. Div., Proc. ASCE, Vol.92, ST-1, Feb. 1966, pp.267-325.

59. Cornell, C.A., 'Bounds on the Reliability of Structural Systems', J. of Struct. Div., Proc. ASCE, Vol.93, ST-1, Feb. 1967, pp. 171-200.

60. Kameda, H., and Koike, T., 'Reliability Theory of Deteriorating Structures', J. of Struct. Div.,Proc.ASCE, Vol.101, No. ST1, Jan. 1975, pp. 295-310.

61. Task Committee on Structural Safety, 'Structural Safety-A Literature Review', J. of Struct. Div.,Proc. ASCE, Vol.98, ST-4, April 1972, pp. 845-884.

62. Turkstra, C.J., 'Choice of Failure Probabilities', J. of Struct. Div., Proc. ASCE, Vol.93, ST-12, Dec.1967, pp. 189-200.

63. Ang, A.H.S., and Amin, M., 'Reliability of Structures and Structural Systems', J. of Engg. Mech. Div., Proc. ASCE, Vol.94, EM-2, April 1968, pp. 671-691.

64. Ang, A.H.S., and Amin, M., 'Safety Factors and Probability in Structural Design', J.of Struct. Div.,Proc. ASCE,Vol.95, ST-7, July 1969, pp. 1389-1405.

65. Shah, H.C., The Rational Probabilistic Code Format', J. of ACI, Vol.66, No.9, Sept. 1969, pp. 690-697.

66. Soxsmith, R.G., and Nelson, M.F., 'Limitations in Application of Probabilistic Concepts', J. of ACI, Vol.66, No.10, Oct. 1969, pp. 823-828.

67. Benjamin, J.R., and Lind, N.C., 'A Probabilistic Basis for a Deterministic Code', J. of ACI, Vol.66, No.11, Nov. 1969, pp. 857-865.

68. Cornell, C.A., 'A Probability Based Structural Code', J. of ACI, Vol.66, No.12, Dec. 1969, pp. 974-985.

69. Lind, N.C., 'Consistent Partial Safety Factors', J. of Struct. Div., Proc. ASCE, Vol.97, ST-6, June 1971, pp. 1651-1670.

70. Ang, A.H.S., 'Structural Risk Analysis and Reliability Based Design', J. of Struct. Div., Proc. ASCE, Vol.99, ST-9, Sep.1973, pp. 1891-1910.

71. Hasofer, A.M., and Lind, N.C., 'Exact and Invariant Second Code Format', J. of Engineering Mechanics Div., Proc. ASCE, Vol.100, No. EM1,Feb.1974, pp.111-121.

72. Ang, A.H.S., and Cornell, C.A., 'Reliability Bases of Structural Safety and Design', J. of Struct. Div.,Proc. ASCE, Vol.100, ST-9, Sep.1974, pp.1755-1769.

73. Lind, N.C., 'Numerical Approximations for Reliability Analysis', Tech. Note, J. of Struct. Div.,Proc.ASCE, Vol.98, ST-3, March 1972, pp.797-803.

74. Ellingwood, B.R., and Ang, A.H.S., ' Risk Based Evaluation of Design Criteria', J. of Struct. Div., Proc.ASCE,Vol.100, ST-9, Sep. 1974, pp. 1771-1788.

75. Ravindra, M. K., Lind, N.C., and Siu, W.,'Illustrations of Reliability Based Design', J. of Struct. Div., Proc. ASCE, Vol.100,ST-9, Sep.1974, pp.1789-1811.

76. Moses, F., 'Reliability of Structural Systems', J. of Struct. Div.,Proc. ASCE, Vol. 100, ST-9, Sep. 1974, pp.1813-1820.

77. Costello, J.F., and Chu, K., 'Failure Probabilities of Reinforced Concrete Beams', J. of Struct. Div.,Proc.ASCE, Vol.95,ST-10, Oct.1969,pp.2281-2304.

78. Warner, R.F., and Kabaila, A.P., 'Monte Carlo Study of Structural Safety', J. of Struct. Div., Proc. ASCE, Vol.94, ST-12, Dec. 1968, pp. 2847-2860.

79. Sexsmith, R.G., 'Reliability Analysis of Concrete Members', J. of ACI, Vol.66, No.5, May 1969, pp. 413-420.

80. Allen, D.E.,'Probabilistic Study of Reinforced Concrete in Bending', J. of ACI, Vol.67, No.12, Dec.1970,pp.989-992.

81. ACI Committee 318, 'Building Code Requirements for Reinforced Concrete (ACI 318-63)',ACI, Detroit, June 1963, p.144.

82. 'CEB-FIP International Recommendations for the Design and Construction of Concrete Structures',Bulletin D' Information, Comite European Du Beton, Vol.72, June 1970.

83. Chandrasekhar, P., 'Reliability Analysis and Design of Prestressed Concrete Beams', Thesis submitted for Ph.D. degree in August 1974 at I.I.T.Kanpur, India.

84. Chandrasekar, P.,and Dayaratnam, P., 'Analysis of Probability of Failure of Prestressed Concrete Beams', Building Science, Vol.10, No.2, July 1975, pp. 161-167.

85. Ranganathan, R., 'Reliability Analysis and Design of-Prestressed Concrete Beams at Different Limit States', Thesis submitted for Ph.D. degree in May 1976 at IIT Kanpur, India.

86. Ranganathan, R., and Dayaratnam, P., 'Reliability Analysis of Prestressed Concrete Beams', J. of the Bridge and Structural Engineer, Vol.8, No.2, June 1978, pp.11-24.

87. Moses, F., and Kinser, D.E., 'Optimum Structural Design with Failure Probability Constraints', J. of AIAA, Vol.5, June 1967, pp.1152-1158.

88. Moses, F., and Stevenson, J.D., 'Reliability Based Structural Design', J. of Struct. Div., Proc. ASCE, Vol.96, ST-2, Feb.1970, pp.221-244.

89. Rao, S.S., 'Minimum Cost Design of Concrete Beams with a Reliability Based Constraint ', Building Science, Vol.8, No.1, March 1973, pp. 33-38.

90. Chandrasekar, P., and Dayaratnam, P., 'Optimization of Prestressed Concrete Beams with Reliability Constraints', J. of Structural Engineering, Vol.7, No.1, April 1979, pp. 1-6.

91. IS-3495 (Part II-1976), 'Methods of Tests of Burnt Clay Building Bricks , Part II, Determination of Water Absorption', I.S.I., New Delhi.

92. IS-3495 (Part I-1976), 'Methods of Tests of Burnt Clay Building Bricks , Part I, Determination of Compressive Strength', I.S.I., New Delhi.

93. Benjamin, J.R., and Cornell, C.A., 'Probability, Statistics and Decision for Civil Engineers', McGraw Hill, 1970.

94. Kennedy, J.B., and Neville , A.M., 'Basic Statistical Methods for Engineers and Scientists', Second Edition, Harper and Row Publishers, New York, 1976.

95. ASTM 270-1968, 'Standard Specifications for Mortar for Unit Masonry', ASTM Standards for Building Codes, American Society of Testing Materials, Philadelphia.

96. Ang, A.H.S., and Tang, W.H., 'Probability Concepts in Engineering Planning and Design', Vol.1, John Willey and Sons, New York, 1975.

97. Dayaratnam, P., and Ranganathan, R., 'Statistical Analysis of Strength of Concrete', Building and Environment, Vol.11, 1976, pp. 145-152.

98. Kendall, M.G., and Stuart, A., 'The Advanced Theory of Statistics', Vol.2, Charles Griffin and Co., London,1961.

99. Struges, H.A., 'The Choice of a Class Interval', Technical note, J. of American Statistical Association, Vol.21, March 1926, pp.65-68.

100. Mann, H.B., and Wald, A., 'On the Choice of the Number of Class Intervals in the Application of the Chi-square Test', The Annals of Mathematical Statistics, Vol.13, 1942, pp. 306-307.

101. Gumbel, E.J., 'On the Reliability of Classical Chi-square Test', Annals of Mathematical Statistics, Vol.14,1943, pp. 253-263.

102. Williams, C.A., 'On the Choice of Number and Width of Classes for the Chi-square Test of Goodness of Fit', J. of American Statistical Association, Vol.45, 1950, pp.77-86.

103. Watson, G.S., 'The X^2 Goodness of Fit Tests for Normal Distributions', Biometrica, Vol.44, 1957, pp.336-348.

104. Hamdan, M.A., 'The Number and Width of Classes in Chi-square Test', J. of American Statistical Association, Vol.58, Sep. 1963, pp. 678-689.

105. Dahiya, Ram C., and Gurland John, 'How Many Classes in the Pearson Chi-Square Test?', J. of American Statistical Association,Vol.68, Sep. 1973, pp. 707-712.

106. Smirnov, N., 'Tables for Estimating the Goodness of Fit of Empirical Distributions', Annals of Mathematical Statistics, Vol.19, 1948, pp. 279-281.

107. Massey, F .J., 'The Kolmogorov- Smirnov Test for Goodness of Fit', J. of American Statistical Association, Vol.46, 1951, pp. 68-78.

108. Miller, L.H., 'Table of Percentage Points of Kolmogorov Smirnov Statistics', J. of American Statistical Association, Vol.51, 1956, pp. 111-121.

109. Lilliefors, H.W., 'On the Kolmogorov Smirnov Test for Normality with Mean and Variance Unknown', J.of American Statistical Association, Vol.62,1967,pp.399-402.

110. Ranganathan, R. and Dayaratnam, P., 'Statistical Analysis of Floor Loads and Reliability Analysis', J.of the Bridge and Structural Engineer, Vol.7,No.1, March 1977, pp. 17-29.

111. 'Handbook of Mathematical Functions', Edited by Abramowitz, M., and Stegan, I.A., Applied Mathematical Series .55, National Bureau of Standards, U.S.Dept. of Commerce, June 1964.

112. Naylor, T.H., 'Computer Simulation Experiments with Models of Economic Systems', John Wiley and Sons, 1971.

113. Ellingwood, B., 'Reliability Basis of Load and Resistance Factors for Reinforced Concrete Design', NBS Building Science Series, National Bureau of Standards, U.S. Department of Commerce, Washington D.C., Feb. 1978.

114. Ellingwood, B., 'Safety Checking Formats for Limit States Design', J. of Struct. Div., Proc. ASCE, Vol.108, ST-7, July 1982, pp. 1481-1493.

115. IS 875-1964, 'Code of Practice for Structural Safety of Buildings: Loading Standards', ISI, New Delhi.

116. Personal Communication to Prof. A.W. Hendry, University of Edinburgh, U.K.

117. Rengaswamy N.S., Balasubramanyam, T.M., Venkataraman, H.S. and Rajagopalan, K.S., 'Corrosion of Reinforcement in Reinforced Concrete and Reinforced Brickwork', Indian Concrete Journal, Vol.38, No.6,June 1964, pp.233-237.

118. 'Corrosion of Reinforcement in Reinforced Brick and Reinforced Cement Concrete', Technical Report No. 20, National Building Organisation, New Delhi, June 1969.

119. Dyaratnam, P., 'Durability of Reinforced Brickwork', Int. Seminar/Workshop on Planning,Design,Construction of Load Bearing Brick Buildings for Developing Countries', New Delhi, Dec. 1981, pp. 283-293.

120. Ragagopalan, K.S., Rengaswamy, N.S.,Balasubramanyam, T.M., and Chandrashekharan, S., 'Corrosion of Reinforcement in Reinforced Concrete and Reinforced Brick Constructions', Indian Concrete Journal, Vol.48, No.5, May 1974,pp.163.

121. Bazant, Z.P., 'Physical Model for Steel Corrosion in Concrete Sea Structures - Theory', J. of Struct. Div., Proc. ASCE, Vol.105, No. ST6, June 1979, pp.1137-1153.

122. Bazant, Z.P., 'Physical Model for Steel Corrosion in Concrete Sea Structures-Application', J. of Struct. Div., Proc. ASCE, Vol.105, No.ST6, June 1979, pp.1155-1166.

123. Lea, F.M., 'The Chemistry of Cement and Concrete',3rd Edition, Edward Arnold (Publishers), London,1970.

124. Knöfel, D., 'Corrosion of Building Materials', Translated by Diamant, R.M.E. from German, Von Norstrand Reinhold Company, New York, 1978.

125. Mehta, P.K., and Haynes, H.H., 'Durability of Concrete in Seawater', J. of Struct. Div., Proc. ASCE, Vol.101, No. ST8, August 1975, pp. 1679-1686.

126. Mindess, S., and Young, J.F., 'Concrete', Prentice Hall, New Jersey, 1981.

127. Benningfield, N., 'Aspects of Cement-based Mortars for Brickwork and Blockwork', Concrete, Vol.14, No. 1 ,Jan.1980, pp. 27-30.

128. Figg, J.W., 'Rusting of Reinforcement- The No.1 Problem of Concrete Durability', Concrete, Vol.14, No.5,May 1980, pp.34-36.

129. Evans, U.R., 'The Corrosion and Oxidation of Metals: Scientific Principles and Practical Applications', Edward Arnold (Publishers) Ltd., London, 1960.

130. Roberts, N.P., 'The Resistance of Reinforcement to Corrosion', Concrete, Vol.4, No.10, October 1970, pp. 383-387.

131. Fontana, M.G., and Greene, N.D., 'Corrosion Engineering', 2nd Edition, McGraw Hill, New York, 1978.

132. Shrivastava, S.N.P., and Dayaratnam, P., 'Behaviour of Reinforced Brick Beams Under Pulsating Loads', International Journal of Masonry Construction, Vol.1, No.3, 1981, pp. 98-102.

133. 'Handbook of Chemistry and Physics', edited by R.C. Weast, 60th Edition, CRC Press, Florida, 1979-1980.

134. Hendry, A.W., 'Structural Brickwork', The Macmillan Press Ltd., London, 1981.

135. Rajagopalan, K.S., Chandrasekaran, S., Rengaswamy, N.S., Balasubramaniam, T.M., Chandrasekaran, V., and Muralidharan, V .S., 'Field Exposure Studies on Corrosion of Reinforcing Steel in Concrete', Indian Concrete Journal, Vol.52, No.9, September 1978, pp. 231-240.

136. Hendry, A.W., Sawko, F., and Sutherland, R.J.M., 'Comments arising from the Discussion of Third Seminar on the Theory of Masonry Structures', BCRA, December ,1981.

APPENDIX - A

Y is a function of several random variable $X_1, X_2, \ldots X_n$ is given by

$$Y = g(X_1, X_2, \ldots\ldots\ldots, X_n)$$

By expanding the function around the mean values of the random variable and taking expectations and variance on both sides neglecting the higher order terms, the approximate mean and standard deviation can be computed as (96)

$$Y_m \approx g(X_{1m}, X_{2m}, \ldots\ldots, X_{nm})$$

$$s_Y^2 \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 s_{X_i}^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \left(\frac{\partial g}{\partial X_i}\right) \left(\frac{\partial g}{\partial X_j}\right) s_{X_i} s_{X_j}$$

where the partial derivatives are to be calculated at the mean values of X_i. X_{im} and s_{X_i} are the mean and standard deviation of the variable X_i. ρ_{ij} is the correlation coefficient of the random variable X_i and X_j. If X_i's are independent random variable, variance of Y becomes

$$s_Y^2 \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 s_{X_i}^2$$

APPENDIX - B

Introducing partial safety factors γ_w and γ_y for strengths of masonry and steel respectively, the design moment M_d can be expressed as

$$M_d = A_{st}\frac{f_{yk}}{\gamma_y} \cdot d\left(1-0.59\ \frac{A_{st}}{bd} \cdot \frac{f_{yk}/\gamma_y}{f_{wk}/\gamma_w}\right)$$

$$\text{if } \frac{A_{st}}{bd} \cdot \frac{f_{yk}/\gamma_y}{f_{wk}/\gamma_w} < 0.319 \qquad \text{(B-1)}$$

$$= 0.259\ bd^2 \frac{f_{wk}}{\gamma_w} \quad \text{if } \frac{A_{st}}{bd} \cdot \frac{f_{yk}/\gamma_y}{f_{wk}/\gamma_w} \geq 0.319 \qquad \text{(B-2)}$$

From Eq. B-1 the area of steel A_{st} for an under-reinforced section can be computed from the following quadratic equation

$$A_{st}^2 - \frac{C_2 bd}{0.59\ C_1} A_{st} + \frac{b\ C_2 M_d}{0.59\ C_1^2} = 0 \qquad \text{(B-3)}$$

where

$$C_1 = \frac{f_{yk}}{\gamma_y} = (1 - 1.645\ \delta_{fy}) \cdot \frac{f_{ym}}{\gamma_y}$$

$$C_2 = \frac{f_{wk}}{\gamma_w} = (1 - 1.645\ \delta_{fw}) \cdot \frac{f_{wm}}{\gamma_w}$$

M_d = design bending moment.

The lowest root given by Eq. B-3 gives the required area of steel. Using $\gamma_w = 2.0$, $\gamma_y = 1.15$, area of steel (referred as A_{std}) is computed and is given in Table 5.1.